Yield Determinants and the Role of ESM Loans in the Primary Market for Spanish Sovereign Debt*

Josha van Spronsen† & Roel Beetsma‡

August 13, 2024

Abstract

We investigate the determinants of primary market yields on Spanish sovereign debt, focusing in particular on the role of ESM lending. Using an innovative multi-stage model addressing endogenous factors with machine learning, we find that a positive shift in the risk-free yield curve and an increase in the CDS spread raise primary yields uniformly across the maturities on issue. Higher inflation raises medium- to long-term yields, while longer outstanding maturities raise short-term yields and lower long-term yields. Larger holdings by the Banco de España lower short-term yields and raise medium-term yields. While ESM loans raise yields across maturities, their impact on issuing costs is minimal. Volatility in primary yields increases with the risk-free yield curve and falls with liquidity for short maturities. Central bank holdings lower volatility for medium-term maturities, while ESM debt relief lowers uncertainty at short maturities.

JEL classification: C26, E43, E44, E62, F43, G12, G15.

Keywords: primary market yields, Spanish sovereign debt, ESM lending, machine learning, instrumental variables.

*We thank participants at an ESM seminar, and the King's College ECONDAT 2023 Spring and IAAE 2024 annual conferences for helpful comments. We also thank the Spanish Debt Management Office, in particular Pablo de Ramon-Laca, for kindly providing data. Finally, we thank Kimi Jiang for helping to construct our dataset. ***Disclaimer:*** *The views expressed here are those of the authors and not necessarily those of the institutions the authors are affiliated with.*

†*Corresponding author.* University of Amsterdam, Tinbergen Institute. Email: j.j.m.vanspronsen@uva.nl.

‡University of Amsterdam, European Fiscal Board, CEPR, CESifo and Tinbergen Institute. Email: r.m.w.j.beetsma@uva.nl.

1 Introduction

During the eurozone sovereign debt crisis distressed sovereigns received official loan packages because they were no longer able to place their debt on the international capital market. While secondary sovereign debt market spreads have been extensively researched, this is much less the case for primary market yields, i.e. the yields against which sovereigns issue the debt. Understanding the determinants and behavior of primary market yields is crucial, as these factors directly influence sovereign debt-servicing costs and, in extreme cases, govern market access.

This paper focuses on the determinants of Spanish primary market yields, a particularly compelling case as Spain was the only eurozone country to issue sovereign debt on a large scale while under an official European Stability Mechanism (ESM) lending program. This unique context allows for a thorough investigation of how ESM lending, alongside other factors such as intervention policies by the European Central Bank (ECB), influences primary yields and, by extension, sovereign market access. Additionally, Spain's experience provides a valuable lens through which to assess the broader effects of ESM interventions on sovereign borrowing conditions over time.

Before we turn to the empirical analysis, we present a simple theoretical framework for the pricing of new debt issued by the sovereign to its primary dealers, who have limited risk-bearing capacity due to limited trading wealth. Based on Ghezzi (2012) the framework extends Van Spronsen and Beetsma (2022) by including the potential realization of an adverse economic state leading to default and a price-insensitive credit line provided by the ESM. The model shows that an increase in the size of the ESM loan affects the price of the new debt through three channels. First, it influences the recovery value for primary dealers in the event of a default. The direction of this effect on the bond price is contingent on whether the ESM debt is perceived as senior to the existing debt. Secondly, it affects the variance of the returns, particularly the downside risk. The direction of this impact on the bond price depends on the response of the recovery value to the size of the ESM loan. Lastly, a larger ESM loan reduces the amount of debt that needs to be issued in the primary market, which exerts upward pressure on the bond price.

While the role of IMF official loans has been widely studied,[1] empirical analyses of the consequences of ESM loans are hardly available. Although the European Commission (2016) reviews the Spanish financial assistance program, it lacks an econometric analysis of the determinants of its success. Closer to our work is Corsetti et al. (2020), who show with a simple event study that IMF loans did not affect market access of distressed eurozone countries, while loans from European institutions did. However, the authors only focus on secondary market yields and do not investigate sovereign primary market yields.

We construct a unique data set covering all Spanish capital market tap issues since the start of the euro. Moreover, we use data from the ECB's public sector purchasing programs and the ESM's lending program for Spain. These data allow a fully-fledged exploration of the determinants of primary yields and, hence, of how eurozone macroeconomic policies can aid in containing yields.

The empirical framework is a novel multi-layer model accounting for the potentially endogenous timing and maturity selection of the auctions, in order to isolate the driving factors behind the primary market yields. In particular, we model each auction as the result of three decision stages by the Debt Management Office (DMO). The first stage concerns the choice of the specific auction day. DMOs attach a high value to predictability and this is borne out by our estimates. There is a high degree of regularity in the choice of the auction days, and they are all or almost all determined via the auction calendar available at the start of the calendar year. The second stage concerns the choice of the set of maturities and target amounts to be issued on the given auction day. We find strong evidence that longer-term issues tend to be followed by shorter-term issues, and vice versa. This suggests that the DMO selects the maturities to be issued so as to keep the average maturity of the outstanding debt stable. Moreover, our findings suggest that contemporary market conditions play a minimal to no role in determining the timing and maturity of the debt issuance.

[1] For example, see Reinhart and Trebesch (2016) for a historical overview and Pisani-Ferry et al. (2013) for an early assessment of the effects on the eurozone economy of IMF loans provided in response to the sovereign debt crisis.

The impact of various explanatory variables on primary yields is in line with our priors. Upward shifts in the risk-free yield curve and increases in the CDS spread raise primary yields across all maturities, while higher inflation raises yields for medium- to long-term maturities. A longer average outstanding maturity raises short-term yields but lowers long-term yields, consistent with the presence of local preferred habitat effects (see e.g. Vayanos and Weill (2010)) combined with a policy in which the DMO tries to bring the maturity back to its longer-run average, thereby supplying relatively more short debt. Larger holdings by the Banco de España lower short-term yields and raise medium-term yields, possibly in anticipation of a future unwinding of its holdings. ESM debt relief increases primary yields across a substantial part of the maturity spectrum, reflecting potential perceptions of ESM debt seniority. However, the impact of ESM debt relief on issuing costs is minimal. Volatility in primary yields increases with the risk-free yield curve and falls with liquidity for short maturities. Central bank holdings lower volatility for medium-term maturities, while ESM debt relief lowers uncertainty at short maturities.

This paper draws on different strands of the literature. First, there is hardly any empirical work directly using primary yields. Krishnamurthy (2000) compares the yields on newly-issued bonds with those on a benchmark bond in the secondary market. However, that paper has a very different focus from ours, because it investigates the profitability of trading the spread between these bonds, but not the determinants of the primary market yields. Further, there is an expanding literature on sovereign debt auctions, in particular on "auction cycles", a pattern of rising secondary-market yields before the auction and falling yields after the auction. Fleming and Rosenberg (2007) and Lou et al. (2013) investigate these movements for the United States, while Beetsma et al. (2016) find auction cycles in Italian, but not German, sovereign debt. Sigaux (2018) corroborates the pre-auction price decline for Italian debt. Beetsma et al. (2018) present a theoretical framework for and empirical evidence of spill-overs of auction cycles across eurozone countries. Van Spronsen and Beetsma (2022) discuss how central bank interventions can mitigate auction cycles by absorbing securities placed in the secondary market by primary dealers for hedging ahead of auctions. Albuquerque et al. (2024) quantify the price elasticity of demand at sovereign bond auctions, linking it to bond return volatility and post-auction returns, suggesting it proxies primary dealer risk-bearing capacity and market pressure. Finally, Ray et al. (2024) investigate how preferred habitat considerations affect the consequences of quantitative easing (QE) by the Federal Reserve, isolating demand shocks through preferred habitat channels. They find that QE has significant local effects on the yield curve when markets are disrupted.

Secondly, to the best of our knowledge, we are among the first to empirically apply machine learning and conformal inference (Lei et al., 2018; Vovk et al., 2005) techniques within an instrumental variable framework, building the methodology outlined in van Van Spronsen (2024).While extensive research has addressed been done on the application of machine learning in economics and econometrics, see e.g. Athey and Imbens (2019) for an overview, the application of machine learning methods to address endogenous variation in continuous variables has been explored to a lesser extent. Hartford et al. (2017) introduce a deep-learning architecture to estimate counterfactual airline prices, but their approach uses a neural network for both stages and is less concerned with causal second-stage effects. In contrast, as in Van Spronsen (2024), we employ CatBoost in the initial stage and regularized regression in the subsequent stage for causal analysis. As demonstrated in Van Spronsen (2024), CatBoost performs effectively with limited data, a valuable feature given the limited size of macroeconomic datasets.

A relatively large strand of the literature explores the determinants of secondary-market yield spreads of other EU sovereigns with Germany, as well as the broader implications of the eurozone debt crisis. Altavilla et al. (2014) discuss the financial and macroeconomic effects of the ECB's outright monetary transactions program, providing context for understanding policy interventions in sovereign debt markets. Favero et al. (2010) argue that the effect of liquidity risk may be underestimated if its interaction with systemic risk is omitted. They proxy a systemic risk factor with the difference between swap rates and government bond yields of a maturity corresponding to that of the sovereign yield spread. Liquidity risk is accounted for by including bid-ask spreads. We do not find a significant role for systemic risk in the Spanish primary market yields.

The fourth strand of relevant literature is the work on the role of official loans. Ghezzi (2012) challenges the belief that official loans can push up sovereign bond yields due to official loan seniority and argues that official loans deflate sovereign yields by alleviating repayment pressure through debt relief.

Expanding on this, Corsetti et al. (2019) examine the strategies deployed by European institutions during the eurozone crisis, demonstrating how different lending mechanisms impacted sovereign borrowing costs. Similarly, Arellano et al. (2017) offer a broader perspective on debt crises, including the role of official lending. Our study contributes to this literature by focusing on the impact of ESM financing on primary yields and its implications for sovereign market access, specifically in the context of Spain.

The remainder of this paper is structured as follows. Section 2 summarises how the eurozone debt crisis played out for Spain, thereby also addressing the role of ESM financial support. Section 3 lays out our simple theoretical framework. Section 4 describes the data sources, the construction of the variables, and the rationales for including these in the analysis. Section 5 describes the empirical strategy and presents the results. In Section 6 we calculate the economic significance of the estimated effects. Section 7 concludes the main text.

2 Spain's crisis and ESM financial support

After strong economic growth and a tripling of house prices in the early euro years, the Spanish real estate bubble burst during the Global Financial Crisis. This resulted in a substantial increase in the non-performing loans (NPLs) of Spanish banks, both due to more bankruptcies in the real estate sector and reduced ability to repay mortgages; see Figure 1.

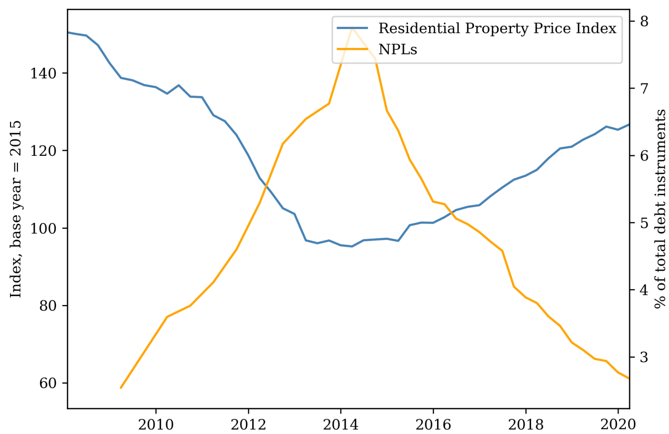


Figure 1: Housing prices and credit performance (NPLs).

Spain, along with the rest of Europe, entered a recession in the first quarter of 2009. As Bentolila et al. (2017) argue, the Spanish economy, with a private sector heavily reliant on banking credit, was further weakened by the credit dry-up in the banking sector, stagnating investment and rapidly-rising unemployment; see Figure 2. Even though the Spanish and eurozone economies improved in 2010 and 2011, Spain entered another recession in the third quarter of 2011 – see Figure 3.

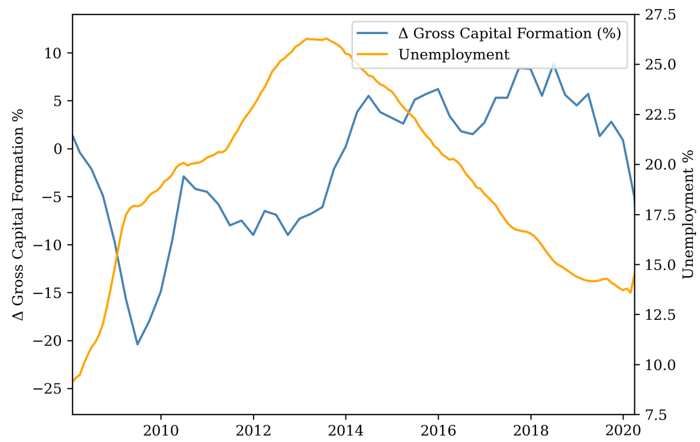


Figure 2: Investment and unemployment.

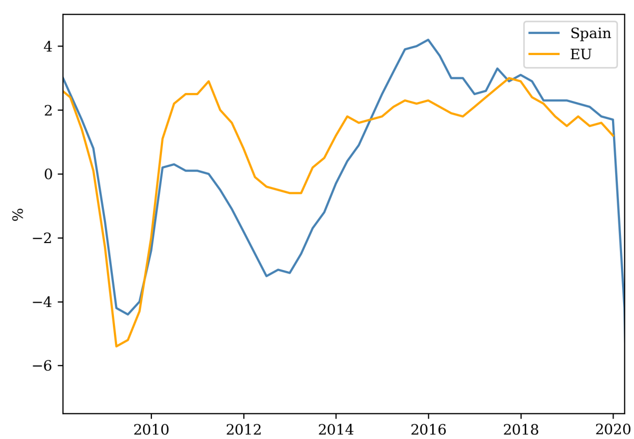


Figure 3: GDP growth (YoY), European Union versus Spain.

Hence, the volume of NPLs continued to increase and the country experienced a substantial increase in the public debt-to-GDP ratio in this period, partly due to rising unemployment and abating tax revenue; see Figure 4.

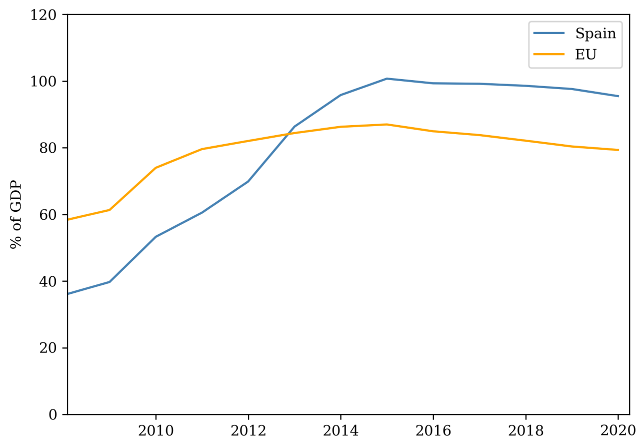


Figure 4: Public debt-to-GDP ratio, European Union versus Spain.

Improving the credit channel by bailing out the banking sector in the presence of dwindling tax revenues and surging sovereign yields (see Figure 5), led to a further deterioration of the government's financial health. Moreover, yield volatility increased sharply. It was against this backdrop that Spain applied for an official loan package from the ESM on June 25^{th} 2012.

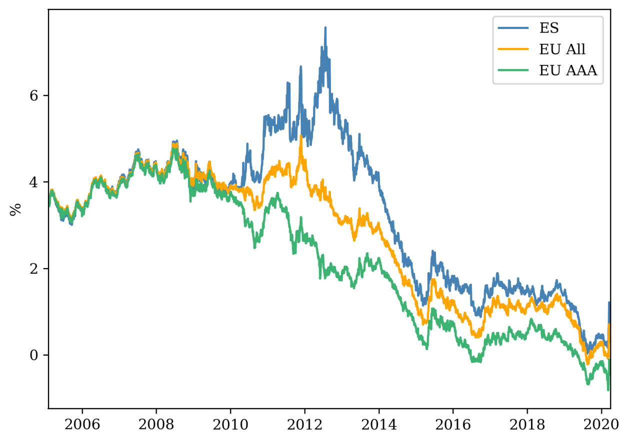


Figure 5: Spanish versus euro area 10Y spot yield.

On December 11^{th} 2012 Spain received its first ESM loan tranche of €39.468 billion, while on February 5^{th} 2013 it received a second tranche of €1.865 billion. The package had a weighted average maturity of 12.5 years and an initial base fee of 0.4855%, over five percentage points lower than the dominant yield in the secondary market. Per the loan conditionality, and at the depth of the recession, Prime Minister Rajoy announced an austerity program of tax increases and public spending cuts on July 11^{th} 2012.

Despite impaired capital market access, Spain never completely lost access to any segment of the capital market during the program. Consequently, it is the only country for which the role of ESM official loans as a determinant of primary market yields can be explored.[2] By implementing structural reforms and diversifying its auction strategy—offering more tenors on the yield curve and multiple issues on an auction day to match investor demand—Spain's economy quickly recovered, allowing the government to regain full capital market access. Spain experienced economic growth in the first quarter of 2014, with declining unemployment and a reduction in non-performing loans

[2] While Greece and Portugal issued bills during their ESM programs, they did not place longer-term debt. The available data for these countries are insufficient in terms of the number of observations and range of maturities to analyze their influence on primary market yields.

(NPLs) starting in the third quarter of 2014; recall Figures 1, 2, and 3. Consequently, Spain began to voluntarily repay its official loans. Table 3 in Appendix C provides an overview of these repayments to date.[3] Figure 6 depicts both the evolution of the outstanding nominal ESM debt, which shows a rapidly declining pattern, and the declining base fee of the ESM debt.

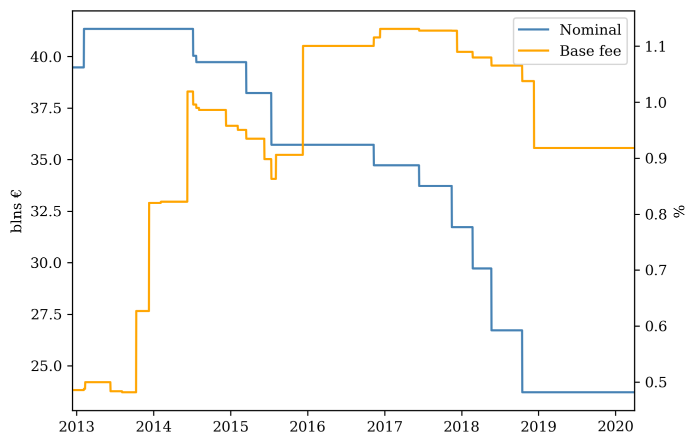


Figure 6: ESM loan: outstanding nominal amount and base fee

3 Description of the theoretical framework

This section outlines a simple theoretical framework underpinning our empirical analysis. The full derivation of the model is detailed in Appendix B. This model, which focuses on primary dealers with risk-bearing capacity – the amount of which is determined by their limited (trading) wealth and their innate risk-aversion – operating in the primary sovereign market, extends that of Van Spronsen and Beetsma (2022). The latter in turn builds upon the asset pricing model in De Jong and Rindi (2009). Our extension introduces the potential realization of an adverse economic state and incorporates a price-insensitive credit line provided by the ESM. The motivation for this assumption is that the objective of its financial assistance programs is

[3]This table is reproduced from ESM data provided at https://www.esm.europa.eu/assistance/spain

not to maximize the expected return (subject to a risk budget). In fact, the ESM is committed to supplying an amount of assets equal to the sovereign's financing need. In the spirit of Diamond and Dybvig (1983), the probability of the economy entering this adverse state can alternatively be interpreted as the proportion of primary dealers who lose confidence in the issuer, thereby demanding a prohibitively high yield or shunning newly-issued debt altogether.

There are N risky assets and one risk-free asset. Further, there are two periods. In period 0, primary dealers use wealth W for trading; in period 1, the asset returns materialize. The gross return on the risk-free asset is $1 + r_f$. There are two possible states in period 1, good (G) and bad (B), materializing with probabilities $1 - \theta$ and θ, respectively, and the returns contingent on the state, $\tilde{F}_G$ and $\tilde{F}_B$, respectively, are normally distributed. Hence, from the perspective of period 0, the gross returns on the risky assets are $\tilde{F} = (1-\theta)\tilde{F}_G + \theta\tilde{F}_B$. Following Ghezzi (2012), the recovery rate in the case of default (the bad state materialising), $\lambda(S_{ESM})$, is a function of the size S_{ESM} of the ESM loan, such that the recovery value equals $\tilde{F}_B = \lambda(S_{ESM})\tilde{F}_G$. Hence, the portfolio value in period 1 is given by $\tilde{w} = X_R^\top \left(1 - \theta\left(1 - \lambda\left(S_{ESM}\right)\right)\right)\tilde{F}_G + \left(W - X_R^\top P_R\right)(1 + r_f)$. Primary dealers maximize expected utility $\mathbb{E}\left[U(\tilde{w})\right]$ with $U(\cdot)$ being the constant absolute risk aversion utility function, leading to a mean - variance optimisation problem.

Denote the supply of bonds on the capital market by S_{CapMkt} and the size of the ESM's official loan by S_{ESM}. Hence, the total amount of debt supplied is $S = S_{CapMkt} + S_{ESM}$. Defining $\eta > 0$ as the coefficient of relative risk-aversion, i.e. $W\kappa = \eta$, assuming market clearing $X_R = S_{CapMkt}$, and assuming without loss of generality that $r_f = 0$, the price against which the risky debt is sold is:

$$P_R = \left(1 - \theta\left(1 - \lambda\left(S_{ESM}\right)\right)\right)\left(\mathbb{E}\left[\tilde{F}_G\right] - \frac{\eta\left(1 - \theta\left(1 - \lambda\left(S_{ESM}\right)\right)\right)}{W}\Sigma_{F_G}\left(S - S_{ESM}\right)\right)$$

where Σ_{F_G} is the variance-covariance matrix of the risky assets. As an example, suppose that following an adverse shock, Spain needs to inject 70 billion euros into the financial sector. Then, S needs to increase by 70 billion. If the ESM provides a loan of 40 billion, the "funding gap" to be placed with primary dealers is 30 billion. The placement puts negative pressure on the price of the bond, hence upward pressure on its yield, while the official credit line of the ESM dampens the increase of the latter. More formally, the impact of a larger official ESM loan on the risky asset prices is given by:

$$\begin{aligned}\frac{\partial P_R}{\partial S_{ESM}} &= \theta \frac{d\lambda(S_{ESM})}{dS_{ESM}} \mathbb{E}\left[\tilde{F_G}\right] \\ &- \frac{2\eta\theta \frac{d\lambda(S_{ESM})}{dS_{ESM}} \left(1-\theta\left(1-\lambda\left(S_{ESM}\right)\right)\right)}{W} \Sigma_{F_G}(S-S_{ESM}) \\ &+ \frac{\eta\left(1-\theta\left(1-\lambda\left(S_{ESM}\right)\right)\right)^2 \Sigma_{F_G}}{W}\end{aligned}$$

The first term represents the effect of the ESM loan on the recovery value to the primary dealer in the event of a bad state. The sign of the derivative of $\lambda(\cdot)$ determines the outcome. If the market perceives ESM debt as senior to the other debt, this derivative will be negative, leading to downward pressure on the price. However, when the recovery value increases with the size of the official loan - potentially due to freed-up fiscal space[4] - this term would be positive, leading to a positive price effect of the size of the official loan.

The second term concerns the change in risk-bearing capacity due to an adjustment in the downside risk. ESM debt influences the variance of the payouts by affecting the recovery rate $\lambda(\cdot)$. Since the primary dealers are risk-averse, this adjustment affects the price of the debt. If the size of the loan has a negative effect on the recovery rate, this reduces the variance of the pay-out, which has an upward effect on the price of the debt. The opposite is the case when the ESM loan raises the recovery rate.

The third term reflects the effect of the reduction in the amount to be placed with the primary dealers in the presence of the ESM loan versus when financing needs have to be met fully in the capital market. The reduction in the amount of capital market financing exerts upward pressure on the bond price.

[4] In his simple model Ghezzi (2012) demonstrates that an increase in the official loan may increase fiscal space.

4 The data

4.1 Primary market data

Our empirical analysis centers on the Spanish sovereign debt market, using a dataset spanning January 2001 to April 2021, which encompasses 974 capital market issuances. We focus specifically on first tranches and tap auctions, deliberately excluding the relatively infrequent syndicated issues. We exclude syndicated issues, because their collaborative nature between the DMO and primary dealers undermines competition and may introduce selection biases on both the supply and demand sides. Addressing these biases would present significant methodological challenges. This resulting dataset comprises 928 auctions, accounting for over 95% of all issues. Our dataset, sourced from Bloomberg, includes information such as ISIN, maturity, average accepted yield, amount bid, amount accepted, and coupon rates. Given the availability of both the issue and redemption dates, maturities are calculated as a year fraction based on the number of business days until maturity, resulting into a continuous variable as opposed to the conventional approach of categorizing issues into discrete maturity buckets. To ensure data accuracy, we cross-validate our dataset with information from the Spanish DMO.

The summary statistics are reported in Table 1. The table reports statistics for each maturity for three sample periods: the whole sample, the crisis period up to Prime Minister Rajoy's austerity program announcement, which we denote by "Pre", and the post-announcement period, which we denote by "Post". The "Pre" period runs from January 1^{st} 2001 until July 10^{th} 2012, while the "Post" period runs from July 11^{th} 2012 until April 30^{th} 2021. For explanatory purposes only we categorize the actual maturities based on their proximity in time to the standard maturities of the buckets.

The 3-, 5- and 10-year maturity buckets are the most frequently issued, over the whole sample and over the second sub-sample. The average accepted yield is upward-sloping in the issued maturity, both in the overall sample and the two sub-periods. While the average of this figure has fallen, its standard deviation has increased between the "Pre" and the "Post" periods. For all maturities, except for the 30-year maturity, the average amount placed at the auctions has fallen slightly between the "Pre" and "Post" periods, as have the standard deviations of the amounts. The fall in the average amounts may be driven by a strategy of playing safe by issuing more but smaller amounts. Finally, the average bid-to-cover ratios tend to decline somewhat with the issuance maturity. There seems to be no systematic difference in their average size between the two sub-periods.

Table 1: Summary Statistics Auctions.

Maturity	N			Av. Acc. Yield (%)			Amount (€blns)			BTC		
	All	Pre	Post	All	Pre	Post	All	Pre	Post	All	Pre	Post
3	213	89	124	1.94	3.31	0.96	1.67	1.83	1.55	2.51	2.29	2.67
				(1.73)	(0.91)	(1.50)	(0.85)	(1.00)	(0.70)	(0.84)	(0.73)	(0.87)
5	257	80	177	1.97	3.84	1.12	1.60	1.83	1.49	2.28	2.20	2.32
				(1.86)	(0.74)	(1.57)	(0.80)	(0.99)	(0.68)	(0.76)	(0.63)	(0.81)
10	243	88	155	2.84	4.63	1.82	1.58	1.79	1.46	2.08	2.25	1.99
				(1.80)	(0.73)	(1.38)	(0.78)	(1.10)	(0.49)	(0.63)	(0.68)	(0.58)
15	91	31	60	3.13	4.89	2.22	1.16	1.31	1.08	2.09	2.60	1.84
				(1.71)	(0.62)	(1.34)	(0.76)	(1.17)	(0.40)	(1.19)	(1.79)	(0.57)
20	32	6	26	2.75	4.73	2.30	0.88	1.15	0.82	1.79	1.70	1.81
				(1.39)	(0.23)	(1.12)	(0.36)	(0.62)	(0.26)	(0.48)	(0.37)	(0.51)
30	83	40	43	3.78	4.96	2.68	1.03	1.09	0.98	1.96	2.30	1.65
				(1.52)	(0.62)	(1.26)	(0.79)	(1.04)	(0.44)	(0.67)	(0.75)	(0.39)
50	9		9	2.32		2.32	1.15		1.15	1.52		1.52
				(0.79)		(0.79)	(0.24)		(0.24)	(0.43)		(0.43)

"Pre" denotes the sample period from January 1st 2001 until July 10th 2012, "Post" the period from July 11th 2012 until April 30th 2021, "Av. Acc. Yield" reports the average accepted yield, "Amount" the amount accepted, "BTC" the bid-to-cover ratio.

4.1.1 Timing of issues

The Spanish DMO establishes its auction calendar at the onset of each year, outlining bid deadlines, settlement dates, and issuance schedules for securities. However, the DMO retains the flexibility to modify this calendar throughout the year based on Treasury funding requirements and market conditions. Adjustments may include canceling or introducing auctions that were not originally scheduled. Such alterations are typically rare, as they could signal adverse market perceptions. Indeed, our analysis indicates no instances of within-the-year deviations from the auction calendar in our sample.

In our dataset, spanning 5305 business days, we observe 413 auction days. Figure 7 illustrates the distribution of time intervals in business days between consecutive auction days, with frequencies normalized to add up to one. Generally, auctions are held every two weeks, and deviations, usually due to holidays, tend to result into a shift of the auction day by one or (rarely) more full weeks. Figure 22 in Appendix D shows that the far majority (close to 90%) of the auction days are Thursdays.

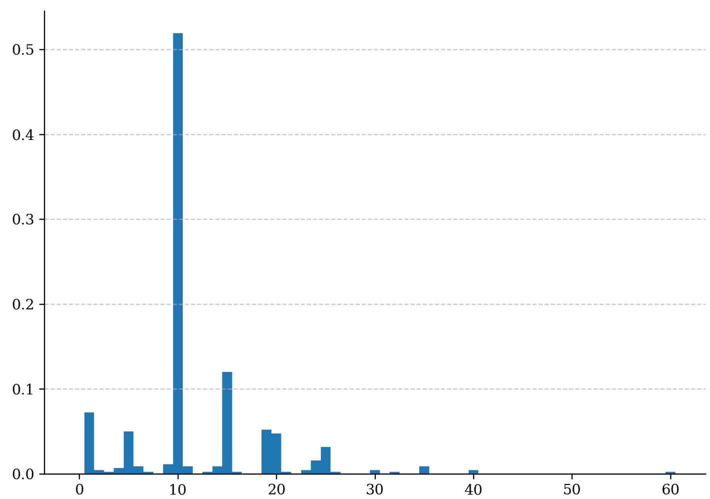


Figure 7: Business days between consecutive auctions, fractions.

Figure 8 presents the monthly count of auction days. Vertical bars in this and subsequent figures mark significant events, including the collapse of Lehman Brothers, the Greek deficit forecast revision, the launch of the ECB's Security Markets Program, the "whatever it takes"-speech (WIT-speech) by former ECB-president Draghi, the Spanish ESM loan, the start of the ECB's Public Sector Purchase Programme, the onset of the COVID-19 pandemic, and the initiation of the ECB's Pandemic Emergency Purchase Programme. The figure indicates a marginally higher average in the second half of the sample period. This increase is largely attributable to a decline in the occurrence of months with only a single auction day.

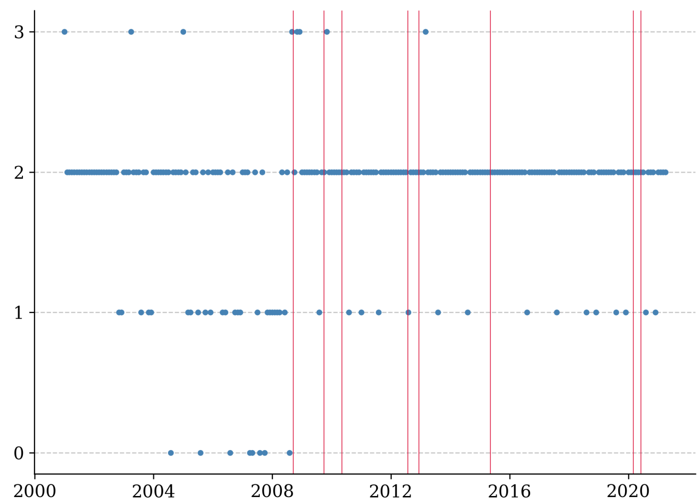


Figure 8: Number of Auction Days by Month.

Figure 9 exhibits a significant rise in the average number of auctions per auction day over time. Initially, the DMO conducted one or two auctions per auction day, which gradually increased to two to three, and subsequently to two to four auctions per auction day. This trend is particularly pronounced following the implementation of the new auction calendar in 2012.

Figure 9: Number of Issues per Auction Day.

Due to the rise in the number of auctions per auction day, the total number of auctions per month has gradually increased over time (see Figure 23 in Appendix D). Prior to the sovereign debt crisis, the Spanish treasury conducted an average of slightly more than two auctions per month (precisely, 2.02). However, following the announcement of the ESM adjustment program, this number increased to an average of approximately five and a half auctions per month. This increase is consistent with the observed rise in the average number of auctions per auction day.

4.1.2 Issue characteristics

Despite the crisis and subsequent austerity measures, the timing of auction dates has remained unchanged, but the issue characteristics have shifted significantly. Our data indicate that the DMO frequently adjusts its outstanding portfolio, to steer the average maturity of its outstanding debt. Figure 10 illustrates the weighted average outstanding maturity twice: once based on own calculations derived from primary market issues, and once based on data from Banco de España. Several observations stand out. First, our calculations exhibit discrepancies at the sample's start due to limited data availability, resembling a 'burn-in' period for our statistic. However, after approximately a quarter of our sample period, our statistic aligns very closely with Banco de España's data. Nonetheless, our approach, which tracks the portfolio from issue to issue rather than at, say, quarterly intervals, offers a more detailed perspective. Our calculations reveal a distinct saw-tooth pattern, indicating that after placing a relatively long maturity, the DMO makes opposing adjustments in subsequent auctions to maintain the desired outstanding maturity level. Indeed, Figure 11, showing the average maturities over the issues per auction day, demonstrates a wide range of maturity placements, indicative of active maturity management.

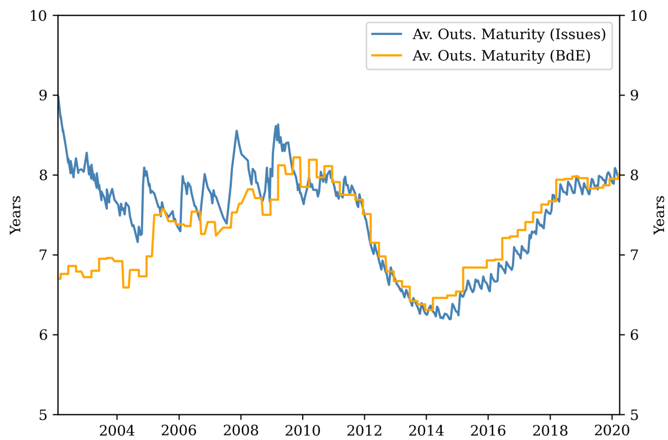


Figure 10: Weighted Average Oustanding Maturity, own calculations based on primary market issues versus Banco d'España data.

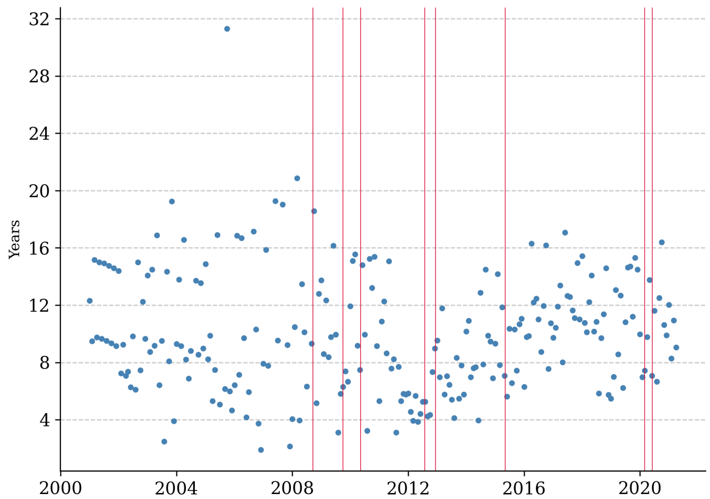


Figure 11: Average Maturities by auction day.

Figure 12 shows the DMO's total targets by auction day, reflecting the Spanish Ministry of Finance's financing needs plus potential rollovers of maturing issues.[5] The DMO only provides the total targets per auction day, not the targets of individual issues. Therefore, aside from single-issue auction days, we lack information on the targeted amounts of individual issues. The total targets by auction day exhibit an upward trend, particularly accelerating after the onset of the sovereign debt crisis. From that moment, despite the rise in the total targets by auction day, the increase in the average number of

[5]Since the DMO only provides total target amounts only from 2010 onwards, we backcast the series to the start of our sample. This task is relatively straightforward for two reasons. First, although the DMO can deviate from the targeted amount depending on the conditions of the moment, the total targeted amount as of 2010 is very close to the total average amount accepted, averaging 266 million euros above the total target, as shown in Figure 24 in Appendix D. Highly favourable conditions may lead to a placement larger than the targeted amount and, vice versa, under sufficiently adverse conditions. Second, until 2010, most auction days featured only one issue, making the average amount accepted almost equal to the target, adjusted for market conditions. We use a CatBoost model with auction outcome data and market and economic variables described in Subsection 4.2 below. Using 2010 as validation holdout sample, the model attains a mean absolute error of 217.5 million euros, approximately 7.3% of the average issue size in 2010; see Figure 25 in Appendix D. Moreover, this error is primarily driven by the very small and large targets at the start of 2010, with the absolute error stabilizing at 6.2% thereafter.

issues per auction day has led to a decrease in the average amount accepted per issue, as shown in Figure 26 in Appendix D.

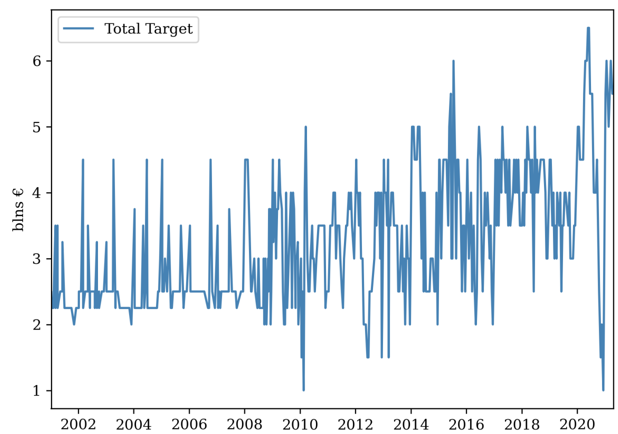


Figure 12: Total targets per auction day.

Further insight into the auction characteristics by month within the calendar year is provided in Figure 27 in Appendix D. Clearly, compared to other months, in August the DMO consistently places less debt, and with shorter average maturity. This pattern persists across both subsamples, indicating a consistent approach. However, while before the sovereign debt crisis the average issue size exhibited more of a U-shaped pattern over the months within a year, in the post-WIT-speech period the average issue size tends to decrease as the calendar year progresses. A potential explanation for this trend is a DMO strategy of securing a significant portion of its total placement target earlier in the calendar year, thereby reducing funding risks.

4.2 Covariates and channels

We collect an array of covariates related to secondary markets, primary dealers, the economic outlook, fiscal policy, public debt, monetary policy, and official loans from diverse sources. To mitigate potential endogeneity concerns arising from simultaneity or reverse causality, we link the Spanish market to risk-free benchmarks proxied by AAA-rated eurozone countries. However, for counterparty-specific covariates, such as default risk, we rely on Spanish statistics.

Secondary market variables

The secondary market indicators are obtained from Bloomberg at the daily frequency or from own calculations based on these data:

- *PD wealth.* The risk-bearing capacity of primary dealers (PDs) is positively correlated with their wealth. Higher PD wealth results into a reduction of the premium they demand on new issues, which subsequently lowers the primary market yield. Although a potential approach could be to use trading capital as a proxy for dealer wealth, the lack of daily data and concerns about its appropriateness — due to the possibility of internal capital transfers between different branches of financial institutions — make this less ideal. Consequently, we measure PD wealth through the market capitalization of the PD's institution, following the methodology proposed by He and Krishnamurthy (2013) and He et al. (2017). Tracking equity returns over the last month provides insights into changes in PDs' risk-bearing capacity. To this end we source from daily equity returns on PDs from Bloomberg.[6]

- *European AAA secondary market volatility*: Risk-averse PDs demand a larger safety premium in more uncertain markets, resulting in a higher primary market yield. We calculate a 10-business-day rolling standard deviation of the European AAA 10-year secondary market yield.

- *Sovereign credit risk.* Credit default swaps (CDSs) offer insights into the market's perception of a default probability. When investors anticipate a higher likelihood of sovereign default, expected bond returns fall and the demand for them weakens. Hence, PDs will find it harder to place new issues with their customers, leading to an increase in primary yields. To account for credit risk, we use CDS spreads on Spanish 10-year debt, because of the liquidity of this market segment.[7]

[6] The PD base can be found on the Spanish DMO's website or in the AFME PD handbooks.

[7] The CDS sample starts only in 2004, which is the reason why the sample size in the ensuing analysis shrinks when we use this variable in our regressions.

- *European AAA liquidity risk.* Increased market liquidity enhances the possibility for PDs to sell the newly-acquired asset to end users, resulting in a lower primary market yield. We construct a eurozone liquidity measure - to ensure exogeneity - at the daily frequency by taking a weighted average of the bid-ask spreads on 3-, 5-, 10, 15- and 30-year German and French debt with weights based on the relative outstanding amounts of sovereign debt of these countries. Taking note of Favero et al. (2010), we limit ourselves to these countries as we confine ourselves to AAA-rated eurozone countries with substantial public debt outstanding to mitigate additional risks, such as systemic risks, which could influence liquidity.

- *Interbank funding conditions.* We utilize the LOIS EUR, which measures the spread between Euribor 3M and EUR SWAP (EONIA) 3M, to gauge credit risk in the euro money market. Tightened funding conditions for primary dealers reduce their capacity to conduct market transactions efficiently, thereby increasing the costs associated with acquiring bonds in the primary market. As the demand for auctioned assets declines, primary market yields increase.

- *Equity volatility sentiment.* Implied volatility metrics, such as the VSTOXX (ticker: V2X), capture market sentiments regarding future equity market volatility. Elevated values of volatility indices often prompt investors to seek refuge in safer assets like sovereign bonds ("flight-to-safety"). Consequently, in anticipation of heightened future equity market volatility, one would anticipate a decrease in primary market yields due to an increased demand for sovereign bonds.

Public debt related variables

Public debt-related data are collected from the Banco de España and the ECB's Statistical Data Warehouse (SDW).

- *Debt-to-GDP ratio.* Higher public debt ratios raise default risk, ceteris paribus, and, hence, lower PD demand for newly-issued debt. Therefore, we expect primary yields to increase with the Debt-to-GDP ratio.

- *Redemptions.* We incorporate the amount of Spanish sovereign debt maturing in the month of the auction, a figure known at the month's outset. Large redemptions may impact the number of issues. Additionally, a higher redemption volume in the current month heightens the risk of not being able to roll over debt at a reasonable cost. We expect PDs to demand a higher yield for buying the new debt.

- *Investor base.* The composition of the investor base may influence the pace at which PDs offload newly-acquired debt. For example, purchases of sovereign debt by national central banks through asset purchasing programs may facilitate the disposal of newly-issued debt, potentially reducing primary market yields. A similar effect might be relevant for long-maturity bonds, as institutional investors demand these securities for hedging purposes.

Macroeconomic variables and leading indicators

Various actual and leading macroeconomic variables are deployed.

- *GDP growth.* Higher GDP growth increases the debt repayment capacity and, hence, we expect it to raise demand for newly-issued sovereign debt. Additionally, periods of strong growth likely coincide with increased risk-bearing capacity among the PDs, exerting further downward pressure on primary yields.[8]

- *Oil price.* The crude oil price is a commonly used indicator of industrial or, more widely, economic activity. Monthly Brent crude 1-month forward prices are obtained from the ECB's SDW.

- *Stock market index.* We measure investors' perceived state of the economy by the level of the STOXX50 blue-chip composite index. Daily data is obtained from the Eurostoxx website.

- *Eurozone AAA Yield curve slope.* The slope of the yield curve serves as an indicator of the future trajectory of the economy.[9] In addition, when the yield curve exhibits a steep slope, the DMO may choose to issue more short-term debt. Daily parameters outlining the shape of the AAA eurozone yield curve are published by the ECB.

[8] We explicitly do not control for GDP levels, as this is merely a scaling variable capturing the size of the economy.

[9] For example, see Federal Reserve Bank of New York, The Yield Curve as a Leading Indicator, https://www.newyorkfed.org/research/capital_markets/ycfaq.html, and the references therein.

Monetary policy
Monetary policy revolves around balancing liquidity demand and supply. The data are again from the ECB's SDW.

- *Eurozone AAA yield curve level.* The European yield curve level, represented by the 10Y pillar, serves as a benchmark for market sentiment and risk perception. When the European yield curve level rises, reflecting changes in the savings-investment balance and possibly increased sovereign stress among borrowers of the highest quality, we expect Spanish primary market yields to rise as well.

- *CB sovereign debt purchases.* Increased sovereign debt purchases by central banks shorten the time PDs hold onto newly-acquired debt, consequently reducing the risk they run and, hence, the yield they demand. To quantify this effect, we follow the methodology outlined by Van Spronsen and Beetsma (2022) and calculate a daily proxy of sovereign debt purchases based on monthly changes in sovereign debt holdings by national central banks.[10]

- *CB sovereign debt holdings.* The central bank's sovereign debt holdings are an indicator of the free float on the secondary market. We define the central bank's sovereign debt holdings as the cumulative amount of sovereign debt purchases the central bank has made to date. The CB sovereign debt holdings may cause a stock effect on primary yields.

Official loans
ESM funding is contingent upon compliance with a macroeconomic program, intended to bolster investor confidence in the country's fiscal stability. Drawing on the insights from Ghezzi (2012), we investigate the role of ESM funding, which entails a nuanced trade-off. On the one hand, the ESM funds are expected to alleviate funding pressures, potentially lowering market yields. On the other hand, given its seniority, or perceived seniority, over newly-issued debt, ESM funding could exert upward pressure on primary yields. Detailed information on ESM loans, including lending fees and loan sizes, is publicly available online.[11]

[10] We set the number of blackout days to zero.
[11] https://www.esm.europa.eu/financial-assistance/programme-database

- *ESM debt relief.* We define ESM debt relief as the difference between the ESM lending rate and the prevailing secondary market yield for debt of a similar maturity, multiplied by the total nominal amount of ESM lending. This measure captures the potential impact of ESM lending on sovereign debt dynamics. A decrease in the ESM lending rate reduces interest payments, thereby decreasing the likelihood of default. Conversely, an increase in the principal amount of ESM debt outstanding may raise default risk on the other debt. Therefore, the effect of ESM debt relief on primary yields could go both ways.

5 Empirical results based on econometric and machine learning methods

In drawing inference about the determinants of primary market yields, it is crucial to address potential endogeneity concerns. The DMO aims to place debt at minimal cost and risk, making the settings of the instruments under their control, such as the timing of auctions, the number of issues, selected maturities and issue targets, potentially endogenous. This endogeneity can conceivably influence the DMO's expected primary market yield as long as placements have not yet materialized. Given the DMO's expertise and market influence, it seems reasonable to assume that the final accepted yield correlates with the DMO's expected outcome.

To mitigate this potential endogeneity, we aim to mimic the DMO's decision function, specifically isolating its decisions on the timing, maturity and target amounts. In the following, we assume that the DMO forms yield expectations and bases its decisions on variables typically used in debt sustainability analysis, such as the macroeconomic outlook, repayment structure and market conditions. Recognizing the potential nonlinearity in these decisions, we employ the supervised learning approach developed by Van Spronsen (2024).[12]

[12] This paper addresses the challenge of seemingly weak instruments in instrumental variable (IV) analysis when the data-generating process (DGP) involves nonlinear relationships. It introduces a supervised machine learning algorithm that remains flexible concerning the first-stage specification, improving robustness in estimating second-stage effects. Additionally, the paper demonstrates the effectiveness of non-parametric testing, specifically conformal inference, in assessing first-stage instrument strength across various DGPs.

We divide our empirical framework into three stages, presented in Figure 13, reflecting the presumed hierarchy and chronology of the DMO's objectives and control. First, we model the timing of the auctions. Next, we model the determination of the maturities and their individual targets on auction.[13] Recognizing the consistent rise in the number of issues per auction day over time (recall Figure 8), we break down this stage by initially forecasting the number of issues on a given auction day and then using these forecasts to model the maturities and targets of these issues. This significantly improves accuracy as the DMO has modified its issuance strategy during the sample period by offering a variety of maturity issues on the same day. Finally, we model the issuance yields. This approach allows us to disentangle the complex relationships between the DMO's decision variables and the issuance yield, comprehensively addressing potential endogeneity concerns.

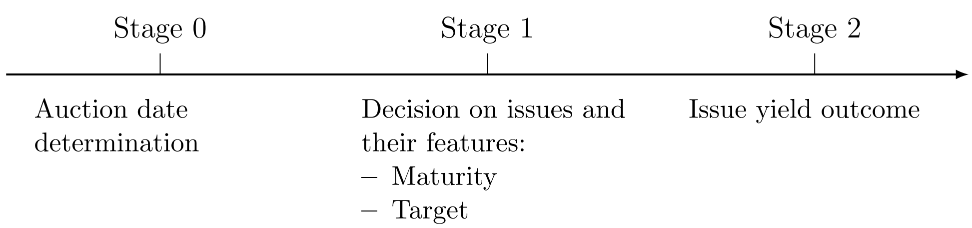


Figure 13: From auction characteristics to outcome

Although the choice of the auction dates is designated a separate stage, our analysis in Appendix E demonstrates that the dates are predetermined by the auction calendar. In other words, auction dates are uncorrelated with other factors, particularly current market circumstances. Therefore, by assuming exogeneity of the auction dates, our framework is reduced to two stages. This enables us to condition the determination of yield on the maturities on auction and their targeted amounts. Henceforth, we limit our sample to the cross-section of all debt issues $i \in \{1, 2, ..., N\}$ and we relabel the time index

[13] Note that while the total targets by auction day are exogenously determined, the DMO retains discretion over the allocation of the targeted amounts across the different maturities issued.

t to refer to the set of unique auction days, i.e. $t \in [1, 2, \ldots, T]$.[14]

In essence, our empirical framework is a two-stage supervised learning instrumental variable (IV) setup, as outlined in Van Spronsen (2024). In this approach, first-stage predictions are generated not by classical methods but by machine learning algorithms, known for their superior predictive performance. Supervised learning enables us to correct for potentially complex nonlinear endogeneity, resulting in stronger first stages – see Van Spronsen (2024) for a formal discussion. In Stage 2, we employ a regularized regression. This setup permits the use of a wide array of covariates without the risk of overfitting the data. By using nonlinearly generated regressors, we obtain prediction-uncertainty-corrected standard errors through non-parametric bootstrapping.

It is important to emphasize that this study focuses on in-sample prediction. Our aim is to filter endogenous variation, similar to standard IV techniques, while accommodating potential nonlinear endogenous effects through a two-stage setup with a nonparametric first stage and nonparametric hypothesis testing. Given the flexibility of the employed methods, there is a higher risk of overfitting compared to standard linear approaches. We mitigate this risk through cross-validation, regularization, and pruning. Consequently, our results reflect generalized in-sample, potentially nonlinear, effects. As discussed in Van Spronsen (2024), this method encompasses standard linear IV approaches, offering comparable statistical power in the absence of nonlinear effects.

5.1 Maturity and target selection

We employ CatBoost, a gradient-boosted regression tree ensemble, to model the issued maturities and targets on specific auction days.[15] Our analysis involves a prediction decomposition process. Initially, we predict the number

[14] Further, we denote the auction day of issue i by τ_i, where $\tau_i \in [1, 2, \ldots, T]$. Note that the elements in the sequence $[\tau_1, \tau_2, \ldots, \tau_N]$ do not have to be unique, since more than one auction may take place on the same auction day.

[15] For a comprehensive explanation of our modeling approach, see Van Spronsen (2024). For detailed information on CatBoost, refer to Dorogush et al. (2018) and Prokhorenkova et al. (2018). CatBoost is an ensemble method, involving gradient-boosting. This includes iteratively adding new trees to the ensemble, with each new tree correcting the errors made by the previous ones. This iterative process continues until a predefined number of

of issues on a given auction day. Then, based on this prediction, we then forecast the maturities and targets to be issued. In Appendix F, we demonstrate that the number of issues to be auctioned can be predicted with high accuracy (96%). This prediction is primarily determined by the total targets on auction days and, to a lesser extent, by central bank debt holdings and the debt-to-GDP ratio.

In addition to the covariates discussed in Subsection 4.2, inspired by the maturity cycling behaviour in Figure 10, our model incorporates the issued maturities and placed amounts of the last five auctions. As suggested in Section 4 the DMO aims at offsetting recent maturity shifts in the outstanding debt, possibly to maintain stability in the average maturity over the shorter run and to only allow for very gradual shifts in the latter over time. Consequently, previous issues are likely to be relevant covariates. Moreover, these variables are expected to satisfy the exclusion criterion and, hence, constitute viable instruments, as they are lagged variables with minimal influence on the contemporaneous auction outcome.

Hyperparameters are optimized using Bayesian optimization (Mockus, 1989), targeting the 5-fold cross-validated root mean squared error. To ensure stability and ease of interpretation, all numerical covariates are transformed to z-scores.[16] Hence, all ensuing estimates, unless otherwise specified, should be interpreted as the effects of a standard deviation increase or decrease. The optimized CatBoost model comprises 300 trees with a maximum depth of 9. At each level it samples 82.95% of the covariates and learns with 7.71%. Each leaf contains a minimum of 15 observations.

trees is reached, or until no further improvement in performance is observed. Each tree is built sequentially, with the algorithm learning from the data to determine the best splits at each level. At each level, CatBoost selects a subset of the features to base the split upon. This helps in reducing over-fitting and improves the generalization of the model. A key feature of CatBoost is its ability to handle categorical variables naturally, without needing them to be one-hot encoded. This helps to preserve the integrity of the data and can lead to more efficient training. Additionally, the algorithm randomly samples a portion of the data for each tree, further enhancing its robustness. The leaves of the trees in a CatBoost model contain predictions for the target variable. During the training, the algorithm adjusts these predictions to minimize a specified loss function, such as the mean squared error or the log loss.

[16]The z-score of a variable X relative to its mean (μ) and standard deviation (σ) is calculated as $z = (X - \mu)/\sigma$.

Following the approach in Van Spronsen (2024), we use conformal prediction (Vovk et al., 2005) to evaluate the strength of our nonlinear first stage. Specifically, as described in Lei et al. (2018), we apply a one-sided Wilcoxon signed-rank test to determine whether an alternative nonlinear model, lacking our instruments, achieves comparable accuracy on the test data.[17] The test rejects the null hypothesis (p-value of 9.26e-8) of the competing model having a smaller error. This outcome indicates that our instruments are indeed relevant and contribute to a robust first stage.

Maturity

Figure 14 presents the scatter plot comparing actual and predicted maturities, categorized by the number of issues auctioned on specific days. The predicted values demonstrate a high accuracy, with a root mean squared error (RMSE) of 1.13.[18] While actual outcomes show clustering of maturities, CatBoost predictions display a more continuous distribution of maturities. Importantly, the algorithm successfully captures the yield curve buckets, with the upper range of predictions aligning with the lower end of the subsequent maturity pillar.

[17] We keep 20% of the data apart to evaluate the model.

[18] On days with one, two, three and four issues auctioned, the RMSEs are 0.87, 1.16, 1.10 and 1.32, respectively. Despite the lowest RMSEs occurring on days with one or three issues —also the most frequent — the RMSEs are comparable across different types of auction days.

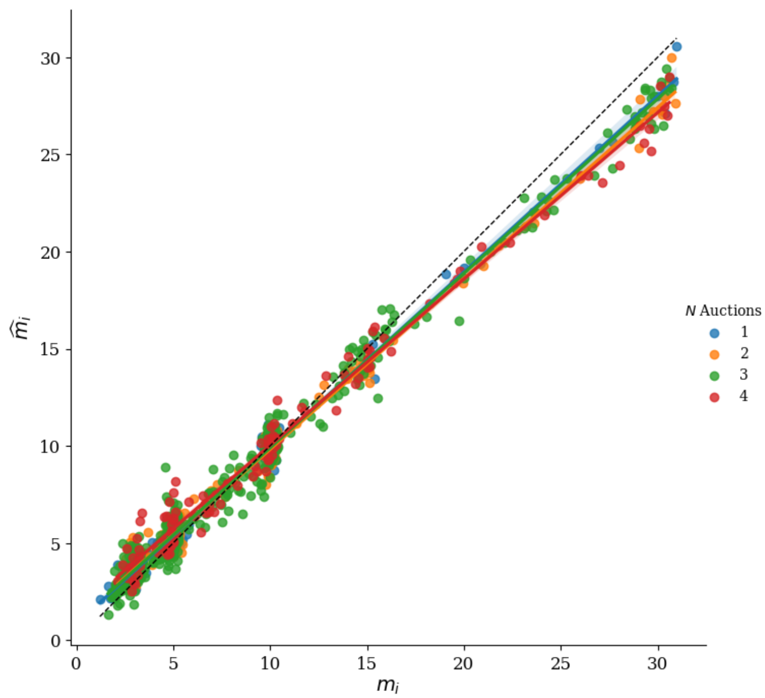


Figure 14: Actual maturities versus CatBoost predictions.

Figure 15 presents the partial dependence plots, highlighting the auction-related covariates. A partial dependence plot (PDP) is a global interpretation method that illustrates the marginal effect of a set of predictor variables, typically one or two for plotting purposes, on the predicted outcome in a machine learning model.[19] These covariates predominantly pertain to the maturity of the outstanding portfolio rather than the prevailing market or economic conditions. Notably, our instruments emerge as among the most relevant covariates in the analysis, which aligns with the observed cycling behavior in the maturity of tap issues, where shorter recent issues are followed by longer current issues. Specifically, the relationship between the maturity of the previous and current issues depends on whether the former exceeds

[19]The sample estimate of the PDP for a single predictor $\mathbf{x}_k$ is defined as $\widehat{f}(z) = \frac{1}{N}\sum_{i=1}^{N}\widehat{f}(\mathbf{x}_i|x_{i,k} = z)$. In other words, the partial dependence function $\widehat{f}(z)$ represents the average prediction of the model across all observations, varying only z, while keeping other predictors fixed at their observed values.

or falls below the long-term average. When the previous maturity exceeds the long-term average, the relationship is negative; conversely, when it is below the long-term average, the relationship is positive. For example, if the previously auctioned maturity is one standard deviation (7.26 years) above the long-term mean (9.06 years), resulting in a maturity of 16.32 years, this reduces the maturity of the current issue by approximately one year. This finding suggests a strategy to continuously keep the average outstanding maturity at a preferred target level, explaining the saw-tooth pattern observed in Figure 10. Further, a momentum or "confidence" effect is discernible in the target amounts: larger placements in previous issues by the DMO correspond to longer maturities chosen in upcoming auctions, although this effect disappears at longer lags. Additionally, the targeted daily amounts exhibit a significant influence on maturity prediction in the form of a nonlinear negative relationship, especially on auction days with relatively low total targets. A possible reason is that it is harder to place relatively large amounts at longer than at shorter maturities.

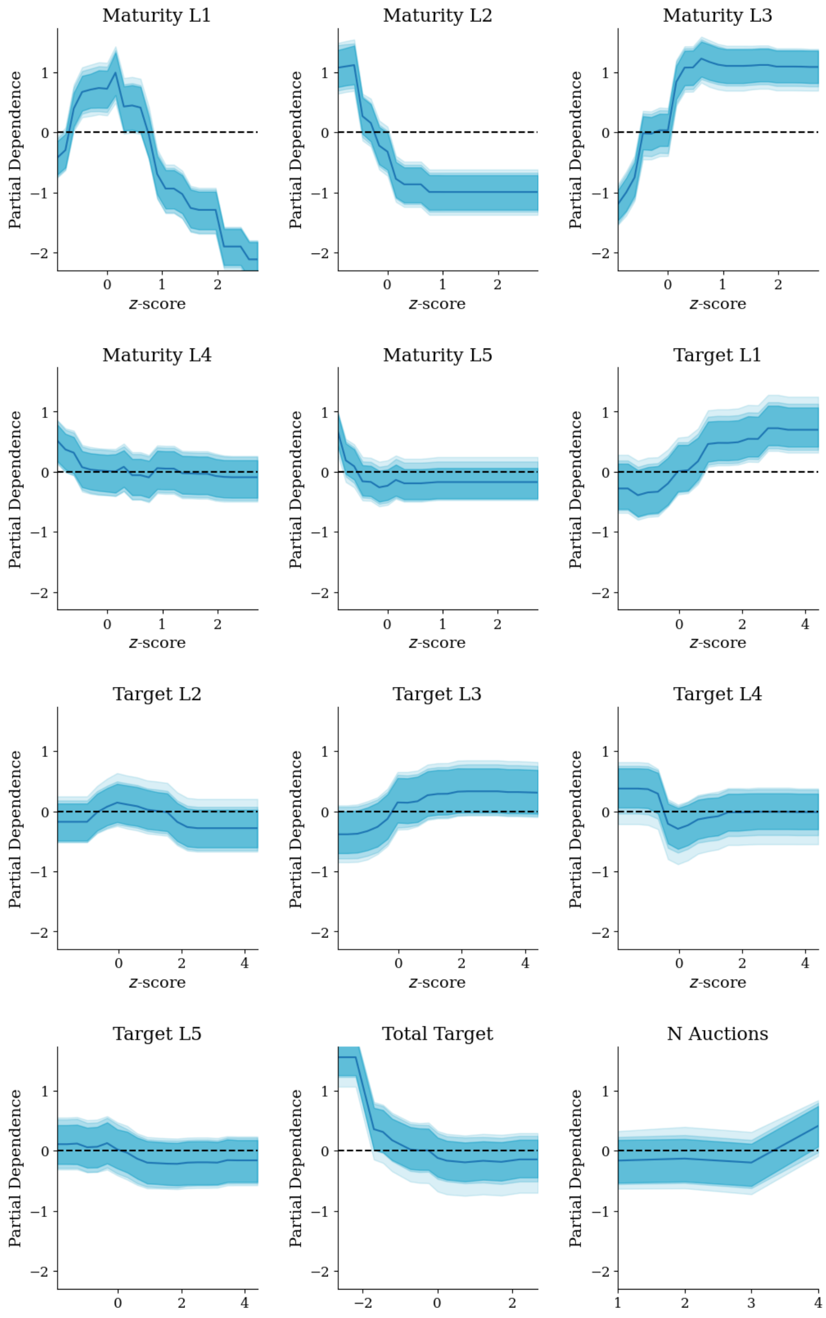


Figure 15: Partial dependence plots of auction related covariates, effect relative to average maturity.

The PDPs of the remaining market- and economy-related covariates, depicted in Figure 28 in Appendix D, exhibit minimal importance of these in predicting the issued maturity. Secondary market volatility exhibits a negative association with the auctioned maturity. This may be due to reduced uncertainty about future interest rate movements during calmer market periods, to which longer-term debt is more sensitive. By issuing longer-term debt during stable market conditions, the DMO can maintain liquidity across all relevant maturities while keeping a check on refinancing risk. Additionally, a decline in oil prices, when they are below average, leads to an increase in the auctioned maturity. Lower oil prices are typically followed by reduced inflation and a more accommodative monetary policy by central banks, which particularly benefits the prices of long-term debt. The DMO may seek to capitalize on this opportunity by issuing longer-term debt.

Target amounts

Figure 16 presents the scatterplot comparing actual and predicted target amounts by auction day, again distinguishing auction days by the number of issues auctioned on these days. Again, the predictions are highly accurate, with a root mean squared error (RMSE) of 0.18.[20] [21]

[20] On days with one, two, three and four issues on auction, the RMSE values are 0.16, 0.21, 0.19 and 0.12, respectively.

[21] Further refinement in predicting targets could potentially be achieved by recognizing that the targets are not continuous but rounded values. However, incorporating ordinal clustering into our multivariate model is non-trivial and, hence, we abstain from this refinement.

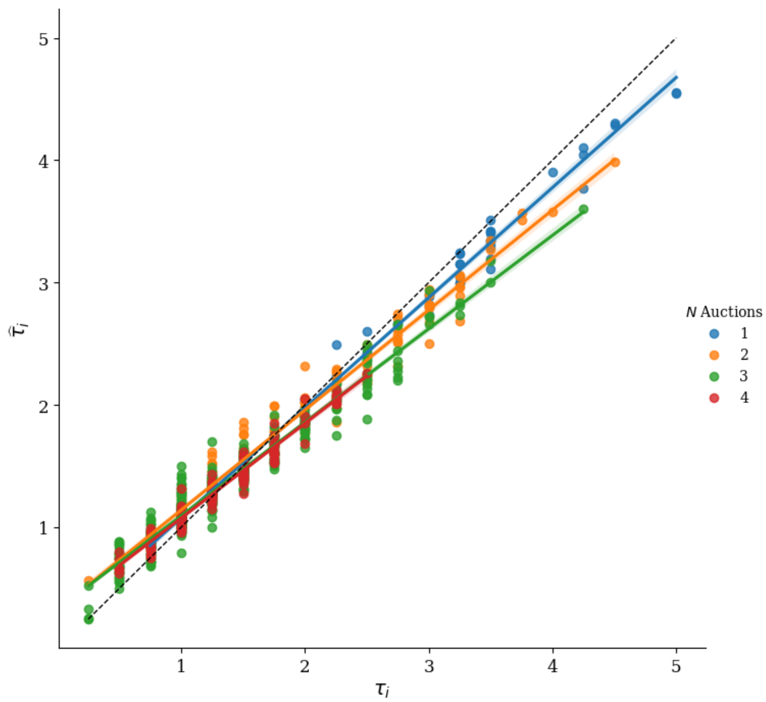


Figure 16: Actual Targets versus CatBoost predictions.

The total daily target emerges as the most relevant covariate, followed by the number of issues on the auction day. Higher total targets naturally lead to higher individual auction targets. However, this relationship levels off at relatively high total targets, where the allocation is typically spread across multiple issues on the auction day. Additionally, the DMO tends to assign larger amounts to earlier issues during a given auction day, potentially to mitigate the risk of falling short of the overall target. Our instrumental variables exhibit less direct relevance, except for the maturity of the first lag. This could be explained by the DMO's strategy of issuing longer-term debt followed by shorter-term issues, which are typically easier to place in the market, and therefore larger.

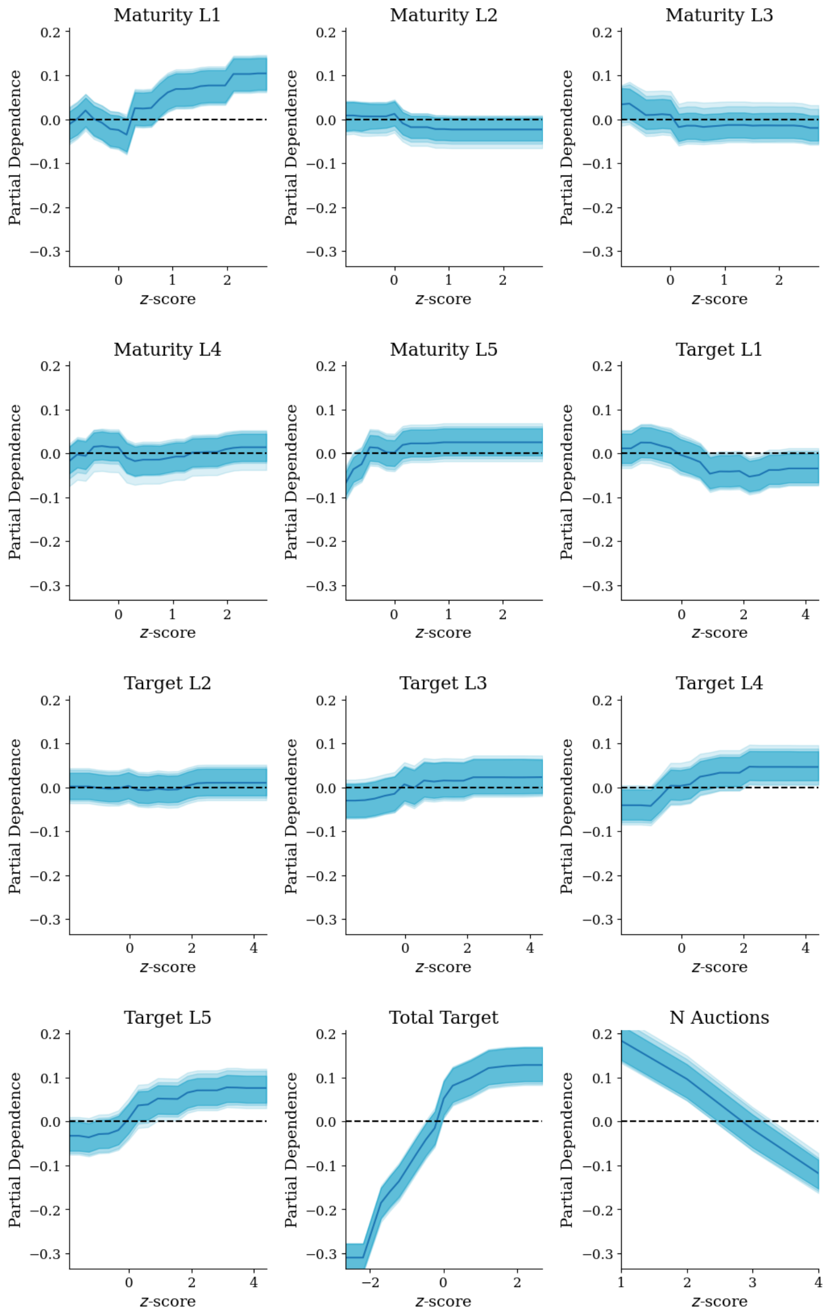


Figure 17: Partial dependence plots of auction related covariates, effect relative to average target.

Regarding the market and macroeconomic variables (see Figure 29 in Appendix D), we observe that a higher yield curve level, slower GDP growth and a steeper yield curve slope are associated with higher targets. Elevated yield curve levels and slower GDP growth may signify increased funding requirements during periods of market stress and rising deficits. A steeper yield curve slope could prompt the DMO to issue relatively larger (and shorter-term) bonds in anticipation of potential interest rate hikes. Additionally, increased central bank purchases predict higher target amounts, likely due to their higher demand which suppresses yield curve yields. Conversely, larger central bank holdings of sovereign debt correspond to smaller issuance amounts, possibly because the smaller free float tends to strengthen price movements from a new issue, see Van Spronsen and Beetsma (2022). The inclusion of year dummies makes timing effects unlikely.

5.2 Primary auction yield

The final stage of our framework concerns the primary yield materialising at an auction, which we model with a regularised regression featuring a dispersion model. Formally,

$$\begin{aligned} y_i &= \left[\left[1, \widehat{m}_i, \widehat{m}_i^2\right] \otimes X_i\right] \beta + \varepsilon_i \\ \varepsilon_i &\sim \mathcal{N}\left(0, \sigma_i^2\right) \\ \sigma_i &= \exp\left(\left[\left[1, \widehat{m}_i, \widehat{m}_i^2\right] \otimes Z_i\right] \gamma\right) \end{aligned} \tag{1}$$

To account for maturity-specific effects of the covariates, we introduce interactions between the predicted maturity and the predicted squared maturity with every regressor in X_i and Z_i. In light of the forbidden regression, the predictions of the squared maturities originate from a prediction model distinct from that for the first-order maturity predictions.[22] To mitigate

[22]This model mirrors its first-order counterpart, except for the inclusion of the squares of the regressors.

multicollinearity and ensure numerical stability, we employ Gram-Schmidt orthogonalisation.

To limit the number of estimated parameters and mitigate potential overfitting, we employ a LASSO penalty (Tibshirani, 1996) and optimize parameter estimation using the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) (Beck & Teboulle, 2009). The optimal penalty parameter is determined via 5-fold cross-validation. Figure 18 displays the cross-validated loss curve, illustrating the relationship between the regularization parameter λ and the corresponding cross-validated loss. Following the widely-adopted "one-standard-error" rule, see e.g. Hastie et al. (2009), we select the most regularized model that maintains a cross-validated loss estimate within one standard error of the minimum, thus ensuring parsimony in the model selection process.

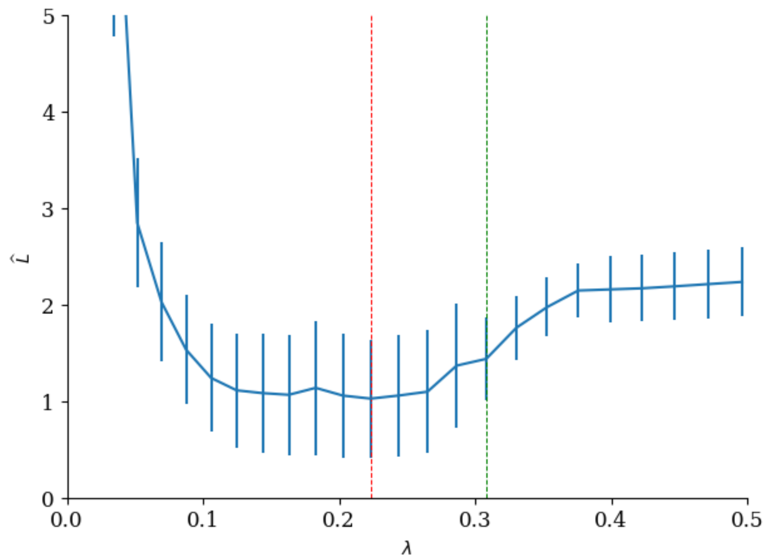


Figure 18: Cross-validated LASSO loss

Figure 19 depicts the estimates of maturity-specific first-order effects, $\widehat{\beta}$, with their bootstrapped confidence intervals.[23] Notably, although the model is capable of identifying quadratic effects, the data indicate that optimal fits can

[23]We only show significant effects. All estimates can be found in Appendix D.

in some cases be achieved with linear curves, in particular for the CDS spread or ESM debt relief effects. Recall that the independent variables are standardized, whereas the dependent variable is not. Therefore, the estimates should be interpreted as the effect in percentage points resulting from a one-standard-deviation increase in the independent variables.

The intercept estimate depicts the estimated primary market yield curve. A higher outstanding maturity of the Spanish sovereign debt portfolio exerts a positive effect on primary yields on short debt and a negative effect on primary yields on longer debt. This can be explained by the DMO's policy of keeping the average maturity at its target. A longer average maturity leads it to issue more short-term debt. In a preferred habitat setting limiting the degree of arbitrage among maturities, this would push up short primary yields and and push down those on longer-term debt.

The relationship between the secondary AAA eurozone yield curve level and primary market yields in Spain exhibits a quadratic pattern. Not surprisingly, in view of the integration of eurozone bond market, an upward shift in the risk-free yield curve causes an upward shift in the Spanish primary yield curve. The effect is strongest for maturities between 15 and 20 years and weaker at the short and very long end. A one-standard-deviation (i.e., 1.41) increase in the European AAA yield curve level corresponds to an approximate rise of 70 basis points in Spanish primary market yields for the short end, one percentage point for the 10-year pillar, and 80 basis points for the long end of the curve.

An increase in the volatility of the eurozone AAA yield curve level leads to higher short-term primary yields, where the shortest maturity exhibits roughly a 5 basis point increase for each standard deviation rise in volatility. This is in line with our theoretical framework which suggests that risk-averse primary dealers demand a higher risk premium in response to increased market volatility. Conversely, medium-term issue yields experience a roughly equal decrease in response to increased volatility. Higher volatility likely encourages the Debt Management Office (DMO) to tilt issuing activity from longer to shorter maturities. Such a shift tends to reduce primary yields on longer maturities at the expense of higher yields on shorter maturities.

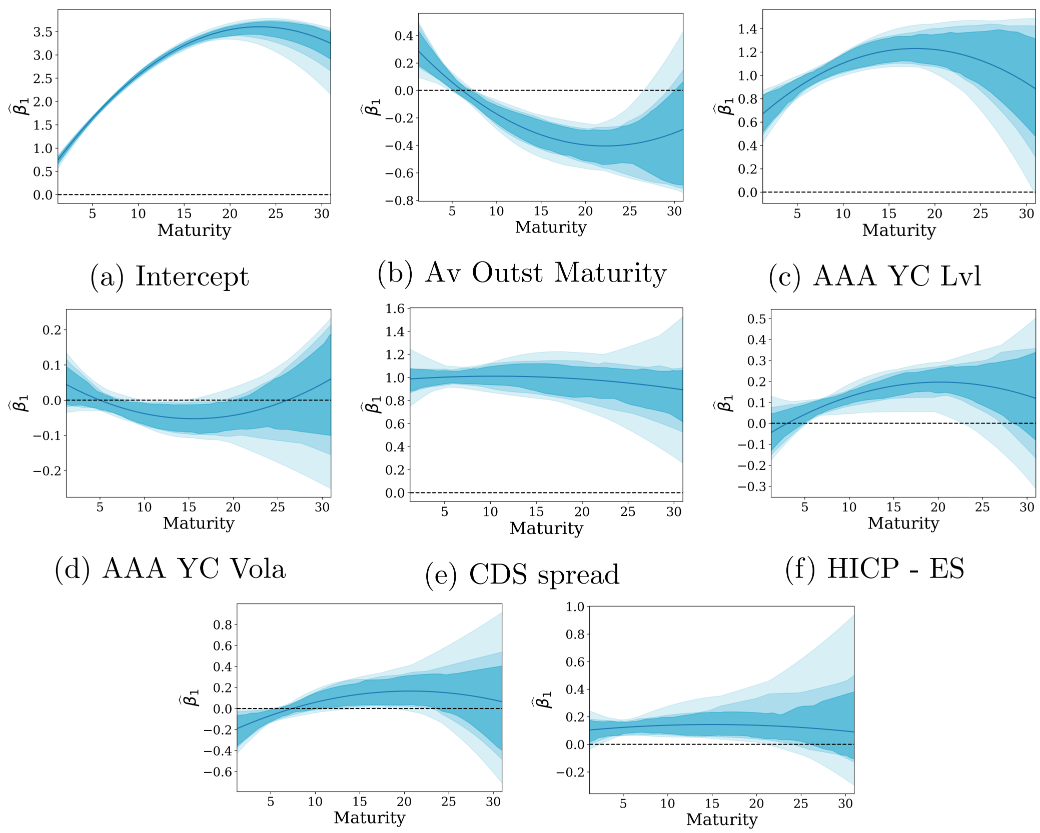
Elevated contemporaneous credit risk, as captured by the Credit Default

Swap (CDS) spread, uniformly pushes up primary market yields, with a standard-deviation increase approximately adding 1 percentage point to the issue yield. Our theoretical framework shows that an increased (perceived) probability of default affects the expected pay-off and the risk-absorption capacity of primary dealers. Empirically, the former effect appears to dominate the latter effect.

Higher inflation raises bond yields, particularly in the medium term. The impact on long maturity issues might be mitigated by an anticipated mean reversion effect. Inflation erodes the real value of future bond payments, causing investors to demand higher nominal yields to protect their real return. In addition, investors may demand an inflation risk premium to compensate for the higher inflation uncertainty that usually accompanies an increase in the level of inflation. Finally, investors may pivot towards assets offering inflation protection, such as commodities or equities. To entice investors back to bonds, issuers need to offer higher yields.

ECB purchases, a flow variable, do not have a significant effect on primary market yields. However, the stock effect represented by the central bank's holdings of sovereign debt negatively influences short-term maturity yields, while it simultaneously increases yields on medium-term maturities. The reason for the latter effect is not immediately apparent. It could suggest pessimistic market sentiments regarding the long-term implications of the quantitative easing programs.

Finally, while ESM assistance may signal a commitment to austerity, structural reforms, and stabilization — benefiting the country in the long run — these conditions can trigger adverse effects on economic growth, social stability and political sentiment in the short term. ESM involvement underscores the economic challenges facing Spain, which investors may interpret as heightened risk or instability of the country's economy. Even under these conditions with ESM support, total debt has surged. Consistent with our theoretical framework, this may explain why primary dealers have demanded higher yields during the period with ESM involvement. Moreover, as also suggested by our framework, even though the ESM debt was issued pari passu, its perceived seniority may exert an additional positive effect on primary yields.



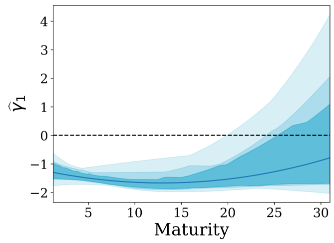
(a) Intercept (b) Av Outst Maturity (c) AAA YC Lvl

(d) AAA YC Vola (e) CDS spread (f) HICP - ES

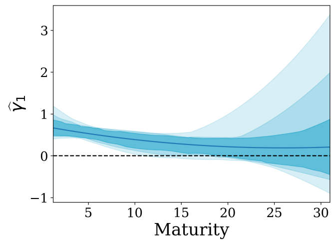
(g) Holdings - BdE (h) ESM debt relief

Figure 19: First-moment effects $\widehat{\beta}$

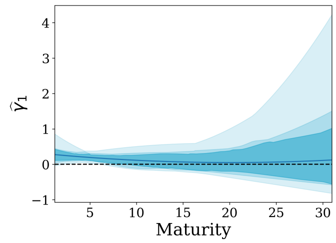
Figure 20 depicts the estimated effects on maturity-specific yield volatility, $\widehat{\gamma}$, along with their bootstrapped confidence intervals. Short- and medium-term primary yields appear to exhibit slightly lower volatility compared to long-term primary yields. An increase in the level of the risk-free yield curve raises volatility of the short and medium term yields. The impact of eurozone market illiquidity mirrors that of the yield curve level, although the effect is smaller. This effect is likely influenced by the challenges faced by primary dealers in selling newly acquired debt during periods of reduced market liquidity, particularly affecting shorter-term securities. Additionally, the central bank's sovereign debt holdings contribute to a reduction in the volatility of short- and medium-term primary yields. Finally, ESM debt relief lowers the volatility of short-term primary yields.



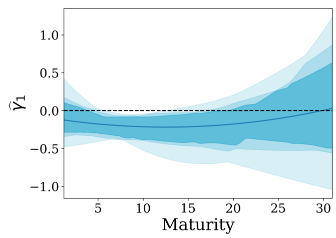
(a) Intercept



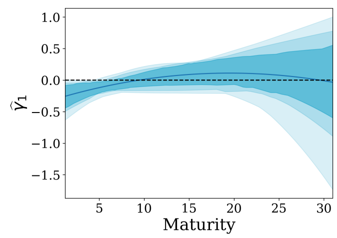
(b) AAA YC Lvl



(c) AAA Illiq



(d) Holdings - BdE



(e) ESM debt relief

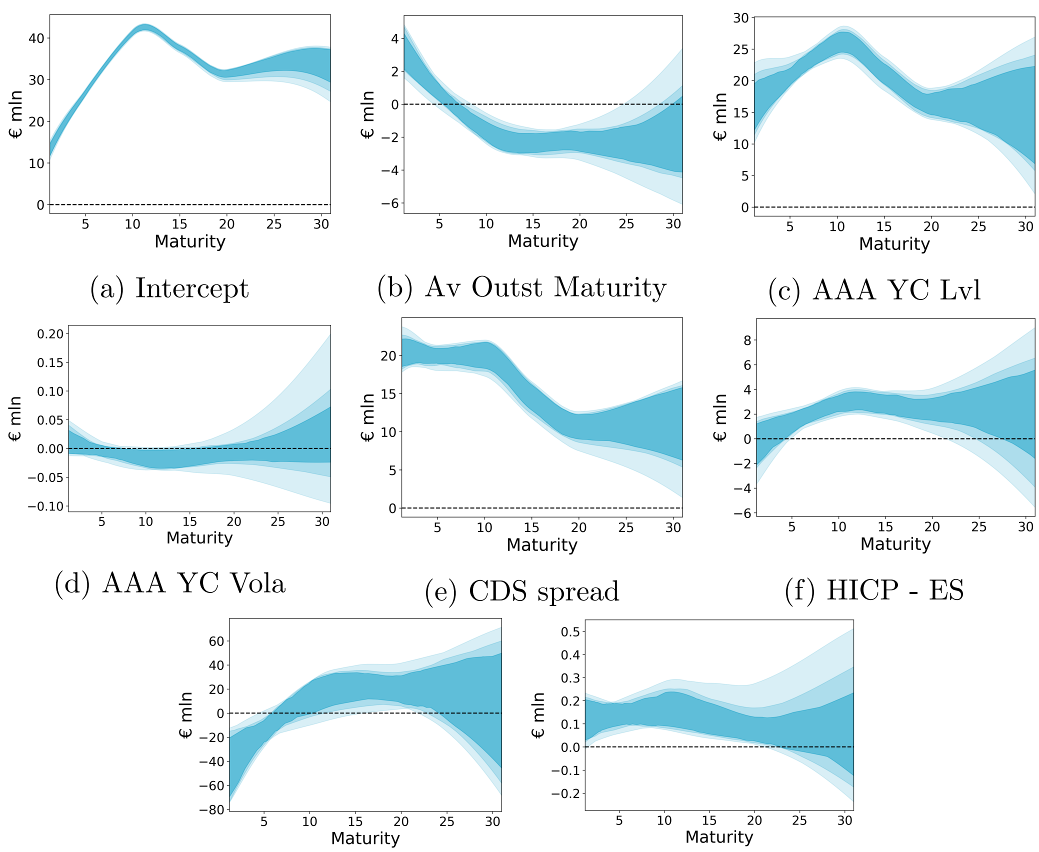
Figure 20: Volatility effects $\widehat{\gamma}$

6 Economic significance

In this section, we aim to assess the economic importance of our findings using a back-of-the-envelope calculation. Given that our estimates are based on standardized variables, we first multiply the estimates by the standard deviation of the respective variable. The resulting numbers are then multiplied by the average issue size across different maturity buckets, as detailed in Table 1. More formally, denote the marginal effect of a one-unit increase in variable X on the cost of a maturity m issue by $c_{m,X}$. Further, denote the sample standard deviation of X by $s(X)$, and the average size of a maturity m issue by p_m. Then, the estimated marginal cost of a one-unit increase in variable X on the annual debt interest payment on a new issue can be written as:

$$c_{m,X} = \widehat{\beta}_{m,X} \times s(X) \times p_m$$

Figure 21 depicts the estimated marginal costs in billions of euros. While all cost effects are generally statistically significant, financially significant impacts are observed only for one-standard deviation movements in the yield curve level, the credit default swap (CDS) spread, central bank sovereign debt holdings and, to a lesser extent, inflation. Notably, there is a minimal effect of ESM debt relief on issuance costs. In addition, variations in issue sizes across the maturity curve contribute to a more uniform distribution of marginal costs across issue sizes of different maturities than indicated by the point estimates alone.



(a) Intercept (b) Av Outst Maturity (c) AAA YC Lvl

(d) AAA YC Vola (e) CDS spread (f) HICP - ES

(g) Holdings - BdE (h) ESM debt relief

Figure 21: Estimated yearly marginal increase in debt service.

7 Concluding remarks

This paper has explored the determinants of Spanish public debt issuance yields. The case of Spain is of particular interest, because Spain was the only country to undergo an ESM loan program while retaining the ability to place longer-term debt on the capital market. The estimation of primary market yield determinants is embedded in a innovative multi-stage decision model, integrating machine learning within an instrumental variables framework.

First, we found that auction dates are entirely predetermined by the auction calendar, independent of contemporaneous market conditions. This allows us to treat auction dates as exogenous. In Stage 1 we modeled the maturities and targeted amounts of the issues. Notably, we observe a cyclical pattern in the maturity of tap issues, where longer maturities tend to be followed by shorter ones, and vice versa, suggesting frequent portfolio rebalancing by the Spanish DMO. In addition, a "confidence" effect is discernible, where larger recent placements are associated with longer maturity issuances. Moreover, higher daily total targets exert a negative effect on the maturities issued. Issue target amounts are increasing in the daily total target, the maturity of the previous issue and the number of issues on the day when this number is not too high. However, the size of the issues falls during an auction day, suggesting a strategy to play by safe and secure the placement of a larger portion of the total target early in the auction day.

In the second stage, we examine the effects of the explanatory variables on primary yield levels and their volatility. The effects on the primary yield levels are quite well in line with what one might expect. While changes in CDS spreads uniformly increase yields, shifts in the risk-free yield curve result in upward adjustments in primary yields that vary according to maturity. Increases in inflation elevate yields across middle to longer maturities. Notably, longer average outstanding maturities raise short-term yields and lower long-term yields, consistent with the observed DMO's cycling strategy to keep the average maturity at its target. Banco de España holdings reduce short-term yields, while they raise longer-term yields. Finally, ESM debt relief elevates yields across various maturities, likely influenced by increased total outstanding debt and a potential perception of ESM debt seniority.

Furthermore, our analysis of primary yield volatility reveals intuitive patterns. Increases in the risk-free yield curve elevate volatility, particularly affecting short- and medium-term maturities. Market illiquidity heightens yield volatility. Enhanced holdings by the Banco de España marginally reduce volatility, predominantly affecting that of medium-term maturities. Notably, ESM intervention mitigates uncertainty in primary markets, especially in the short term.

In terms of their financial significance, shifts in the yield curve, CDS spreads, and the Spanish central bank's sovereign debt holdings exert the most substantial effect on issuance costs. By contrast, the financial impact of ESM debt relief is minimal. Hence, any effect to calm markets through ESM involvement seems to come at little cost.

The findings in this paper provide leads for further research on sovereign debt auctions. An important question is whether our findings are in line with the Spanish DMO following an optimal issuance policy (based on a trade-off of costs and risks) or whether improvements are possible. It is noticeable that the DMO seems to stabilise the average outstanding maturity at a rather high frequency by having longer issues more likely followed by shorter issues, and vice versa. Future studies could explore these dynamics in other countries and investigate the impacts of international financial support instruments like ESM guarantees or IMF relief on primary yields. In addition, examining interactions between primary and secondary markets for identical instruments and the determination of primary yields for new instruments in the market are avenues for future exploration. Here we may a priori expect direct interactions between the DMO and primary dealers to play a substantial role regarding the maturity and other features of the new-to-issue bonds. Finally, an intriguing question is whether the DMO could have retained full market access without official loan support. Our analysis suggests that without this support Spain would not have faced market exclusion. To delve deeper into this matter, exploring how official lending has shaped Spain's public finances and its debt placement policies would be essential.

8 References

Albert, J. H., & Chib, S. (1993). Bayesian Analysis of Binary and Polychotomous Response Data. Journal of the American Statistical Association, 88(422), 669–679 (cit. on p. 73).

Albuquerque, R., Cardoso-Costa, J. M., & Faias, J. A. (2024). Price elasticity of demand and risk-bearing capacity in sovereign bond auctions. The Review of Financial Studies, https://academic.oup.com/rfs/advance-article-pdf/doi/10.1093/rfs/hhae027/58477869/hhae027.pdf. https://doi.org/10.1093/rfs/hhae027 (cit. on p. 4)

Altavilla, C., Giannone, D., & Lenza, M. (2014). The financial and macroeconomic effects of omt announcements. International Journal of Central Banking, 12(3), 29–57. https://www.ijcb.org/journal/ijcb14q3a2.htm (cit. on p. 5)

Arellano, C., Atkeson, A., & Wright, M. L. J. (2017). External and public debt crises. Annual Review of Economics, 9, 317–362. https://doi.org/10.1146/annurev-economics-063016-103616 (cit. on p. 6)

Athey, S., & Imbens, G. W. (2019). Machine learning methods that economists should know about. Annual Review of Economics, 11, 685–725 (cit. on p. 5).

Beck, A., & Teboulle, M. (2009). A fast iterative shrinkage-thresholding algorithm for linear inverse problems. SIAM Journal on Imaging Sciences, 2(1), https://doi.org/10.1137/080716542, 183–202. https://doi.org/10.1137/080716542 (cit. on p. 38)

Beetsma, R., Giuliodori, M., De Jong, F., & Widijanto, D. (2016). Price effects of sovereign debt auctions in the eurozone: The role of the crisis. Journal of Financial Intermediation, 25, 30–53 (cit. on p. 4).

Beetsma, R., Giuliodori, M., Hanson, J., & De Jong, F. (2018). Cross-border auction cycle effects of sovereign bond issuance in the euro area. Journal of Money, Credit and Banking. https://doi.org/10.1111/jmcb.12510 (cit. on p. 4)

Bentolila, S., Jansen, M., & Jiménez, G. (2017). When Credit Dries Up: Job Losses in the Great Recession. Journal of the European Economic Association, 16(3), 650–695 (cit. on p. 7).

Corsetti, G., Erce, A., & Uy, T. (2019). Official sector lending strategies during the euro area crisis. The Economic Journal, 129(617), 214–246. https://doi.org/10.1111/ecoj.12613 (cit. on p. 6)

Corsetti, G., Erce, A., & Uy, T. (2020). Official sector lending during the euro area crisis. The Review of International Organizations, 15(3), 667–705 (cit. on p. 3).

De Jong, F., & Rindi, B. (2009). The microstructure of financial markets. Cambridge, UK, Cambridge University Press. (Cit. on p. 11).

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. Journal of Political Economy, 91(3), 401–419 (cit. on pp. 12, 52).

Dorogush, A. V., Ershov, V., & Gulin, A. (2018). Catboost: Gradient boosting with categorical features support. (Cit. on p. 28).

Elor, Y., & Averbuch-Elor, H. (2022). To smote, or not to smote? https://doi.org/10.48550/arXiv.2201.08528. (Cit. on p. 68)

European Commission. (2016). Evaluation of the financial sector assistance programme. spain, 201 (Institutional Paper 019). Brussels. https://ec.europa.eu/info/publications/economy-finance/evaluation-financial-sector-assistance-programme-spain-2012-2014_en. (Cit. on p. 3)

Favero, C., Pagano, M., & von Thadden, E.-L. (2010). How Does Liquidity Affect Government Bond Yields? Journal of Financial and Quantitative Analysis, 45(1), 107–134 (cit. on pp. 5, 23).

Fleming, M. J., & Rosenberg, J. V. (2007). How do treasury dealers manage their positions? (Staff Report No. 299). Federal Reserve Bank of New York. https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr299.pdf. (Cit. on p. 4)

Ghezzi, L. (2012). Official lending: Dispelling the lower recovery value myth. VoxEU.org. https://voxeu.org/article/official-lending-dispelling-lower-recovery-value-myth (cit. on pp. 2, 5, 12, 13, 25, 52)

Hartford, J., Lewis, G., Leyton-Brown, K., & Taddy, M. (2017). Deep IV: A Flexible Approach for Counterfactual Prediction [ISSN: 2640-3498], In Proceedings of the 34th International Conference on Machine Learning, PMLR. ISSN: 2640-3498. Retrieved February 16, 2024, from https://proceedings.mlr.press/v70/hartford17a.html. (Cit. on p. 5)

Hastie, T., Tibshirani, R., & Friedman, J. (2009). The elements of statistical learning: Data mining, Springer. (Cit. on p. 38).

He, Z., Kelly, B., & Manela, A. (2017). Intermediary asset pricing: New evidence from many asset classes. Journal of Financial Economics, 126(1), 1–35 (cit. on p. 22).

He, Z., & Krishnamurthy, A. (2013). Intermediary asset pricing. American Economic Review, 103(2), 732–70 (cit. on p. 22).

Holmes, C. C., & Held, L. (2006). Bayesian auxiliary variable models for binary and multinomial regression. Bayesian Analysis, 1(1), 145–168 (cit. on p. 74).

Krishnamurthy, A. (2000). The Bond/Old-Bond Spread. https://doi.org/10.2139/ssrn.251769. (Cit. on p. 4)

Lei, J., G'Sell, M., Rinaldo, A., Tibshirani, R. J., & Wasserman, L. (2018). Distribution-free predictive inference for regression. Journal of the American Statistical Asso 113(523), https://doi.org/10.1080/01621459.2017.1307116, 1094–1111. https://doi.org/10.1080/01621459.2017.1307116 (cit. on pp. 5, 30)

Lou, D., Yan, H., & Zhang, J. (2013). Anticipated and repeated shocks in liquid markets. Review of Financial Studies, 26(8), 1891–1912 (cit. on p. 4).

Mockus, J. (1989). Bayesian approach to global optimization: Theory and applications. Springer Netherlands. (Cit. on pp. 29, 70, 75).

Pisani-Ferry, J., Sapir, A., & Wolff, G. (2013). Eu-imf assistance to euro area countries: An early as (Bruegel Blueprint 19 Policy Paper). https://www.bruegel.org/2013/06/eu-imf-assistance-to-euro-area-countries-an-early-assessment/. (Cit. on p. 3)

Prokhorenkova, L., Gusev, G., Vorobev, A., Dorogush, A. V., & Gulin, A. (2018). Catboost: Unbiased boosting with categorical features, In Advances in neural informa (Cit. on p. 28).

Ray, W., Droste, M., & Gorodnichenko, Y. (2024). Unbundling quantitative easing: Taking a cue from treasury auctions. Journal of Political Economy. https://doi.org/10.1086/729581 (cit. on p. 5)

Reinhart, C. M., & Trebesch, C. (2016). The international monetary fund: 70 years of reinvention. Journal of Economic Perspectives, 30(1), 3–28 (cit. on p. 3).

Sigaux, J.-D. (2018). Trading ahead of treasury auctions (Working Paper No. 2208). European Central Bank. https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2208.en.pdf. (Cit. on p. 4)

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. Journal of the Royal Statistic 58(1), 267–288. Retrieved November 4, 2022, from http://www.jstor.org/stable/2346178 (cit. on p. 38)

Van Spronsen, J. (2024). Nonlinear instrumental variables & conformal inference. Working Paper (cit. on pp. 5, 26, 28, 30).

Van Spronsen, J., & Beetsma, R. (2022). Unconventional monetary policy and auction cycles of eurozone sovereign debt. Journal of Money, Credit and Banking, 54(1), 169–202 (cit. on pp. 2, 4, 11, 25, 37, 52).

Vayanos, D., & Weill, P.-O. (2010). Liquidity and the term structure of interest rates. Review of Financial Studies, 23(12), 4786–4825. https://doi.org/10.1093/rfs/hhq046 (cit. on p. 4)

Vovk, V., Gammerman, A., & Shafer, G. (2005). Algorithmic learning in a random world. Springer. (Cit. on pp. 5, 30).

Appendices (NOT for publication; available online or upon request)

Appendix A The European Stability Mechanism

The European Financial Stability Facility (EFSF), the predecessor of the European Stability Mechanism (ESM), was established in 2010 as a temporary vehicle to combat the eurozone's sovereign debt crisis by providing official loans based on conditionality to countries in economic distress. The EFSF had a lending capacity of €440 billion backed by guarantees of eurozone member states. Although the EFSF still exists as a legal entity due to the time length of its programs, it no longer issues new loans.

The ESM was established in 2012 as a permanent entity to safeguard the financial stability of the eurozone. The ESM has €700 billion of subscribed capital, of which €80.6 billion has been paid in by eurozone member states,[24] while the remainder can be raised by issuing ESM bonds ranging from 1- to 45-year maturity.

Upon a request for relief from a country experiencing financial difficulties, the ESM may provide an official loan. However, such a loan will be conditional on the country subjecting itself to a macroeconomic adjustment program based on (structural and other) measures intended to restore the sustainability of its public debt, which would allow it to place new debt on the capital market.[25] As ESM loans are backed by guarantees from its members, lending can be provided against favorable interest rates. As a result, the total relief in debt-servicing costs for countries over the complete ESM program can be as large as 7.5% of GDP.[26]

The EFSF/ESM has been called upon six times. Table 2 summarizes the

[24]As of May 2024, see https://www.esm.europa.eu/esm-governance.
[25]https://www.esm.europa.eu/financial-assistance/lending-toolkit
[26]See https://www.esm.europa.eu/impact-budgets.

main characteristics of each program.[27] As Spain is the only country that both received ESM support and retained full market access during the program, it is the only country available to explore the role of ESM official loans as a determinant of the issuance yields.[28]

Table 2: EFSF/ESM Program Overview.

Country	Cyprus	Greece	Ireland	Spain	Portugal
Start	Apr-13	May-11	Nov-10	Jul-12	May-11
End	Mar-16	Aug-16	Dec-13	Dec-13	May-14
Total size	10	256.6	67.5	100	78
Size ESM	9	203.7	17.7	100	26
Disbursed	6.3	203.7	17.7	41.3	26
Rate	0.9%	1.2%	2%	0.9%	2%
Maturity	14.9	32.35	14	12.5	14
Volunt. repaid	0	16.2	0	24.9	2
Loan Repayments	2025-2031	2034-2060	2029-2042	2025-2027	2025-2040
Market Activity	Syndications 2013	Bills	Syndications 2013	Full	Bills, Syndications 2013-2014

(i) Amounts are in billions. (ii) "Total size" denotes the size of the total rescue package. (iii) "Size ESM" denotes the size of the available EFSF/ESM package. (iv) "Disbursed" denotes the amount actually lent to the country in question. (v) "Rate" denotes the average lending rate. (vi) "Maturity" denotes the size-weighted average maturity. (vii) "Volunt. repaid" denotes the amount of the EFSF/ESM package voluntarily repaid ahead of the maturity date. (viii) "Market activity" states the issuance activity of the Treasury during the EFSF/ESM programs.

[27] As of May 2024, see https://www.esm.europa.eu/financial-assistance/programme-database/programme-overview

[28] Greece and Portugal did issue bills while they were under an ESM program, but they did not place longer-term debt. The available data for these countries are too limited in terms of numbers of observations and spread over maturities to explore their role in driving issuance yields.

Appendix B Model of a distressed bond market and official lending

This Appendix presents the full theoretical model. We present a basic model with primary dealers who trade in the primary market and the ESM, which is price insensitive and provides as much as is deemed necessary to maintain uninterrupted access to the market for sovereign debt. The model builds on Van Spronsen and Beetsma (2022) in which bond purchases by the ESCB are price insensitive.

There are N risky assets and one risk-free asset. Further, there are two periods, period 0 and period 1. In period 0, the representative primary dealer is endowed with wealth W, which it uses for trading in the market. In period 1, the pay-offs of the assets materialize. The risk-free asset pays a deterministic return of $1 + r_f$, while the vector of gross returns of the risky assets $\tilde{F}$ is for analytical reasons assumed to be multivariate-normally distributed. The gross returns are contingent on the state of the world. With probabilities θ and $1 - \theta$, the economy lands in the bad (B) and good (G) state, respectively. We assume that $\tilde{F} = \theta \tilde{F}_B + (1 - \theta)\tilde{F}_G$. The states of the world (B and G) can be interpreted as forming a hidden Markov model. In this framework, F_G denotes the return in scenarios where the debt remains sustainable, while F_B represents the recovery value in the event of sovereign default. Alternatively, in line with Diamond and Dybvig (1983), F_B can also be understood as the proportion of primary dealers who lose confidence in the issuer, consequently adjusting their pricing downward. We assume that the recovery value in case of a default equals the recovery rate $\lambda(S_{ESM})$ times the good state pay-out $\tilde{F}_G$, where, following Ghezzi (2012), the recovery rate is a function of the loan supplied (against favourable conditions) by the ESM. The results below show that the effect the official loan has on sovereign bond prices is driven by the sign of the (perceived) effect that official lending has on the recovery rate. Denote by the vector $X = \left[X_R^\top, x_{rf}\right]^\top$ the demand for the N risky assets X_R and the risk-free asset x_{rf}. The price of the risk-free asset is normalized to unity. We denote the price vector of the risky assets by P_R. Hence, we can write the primary dealer's period-0 budget constraint as $W = X_R^\top P_R + x_{rf}$. The value of the portfolio in period 1 is stochastic and given by $\tilde{w} = X_R^\top \tilde{F} + \left(W - X_R^\top P_R\right)(1 + r_f) = X_R^\top \left(1 - \theta\left(1 - \lambda\left(S_{ESM}\right)\right)\right)\tilde{F}_G + \left(W - X_R^\top P_R\right)(1 + r_f)$. The primary dealer maximizes expected utility $\mathbb{E}[U(\tilde{w})]$ over the portfolio weights, where the function $U(\cdot)$ is strictly concave and twice continuously differentiable. Applying Stein's lemma to the standard first-order conditions,[29] and using some simple algebra, yields:

$$\begin{aligned}\mathbb{E}\left[U'(\tilde{w})\frac{\partial \tilde{w}}{\partial X_R}\right] &= 0 \iff \\ \mathbb{E}\left[U'(\tilde{w})\left[(1-\theta(1-\lambda(S_{ESM})))\tilde{F}_G\right]\right] &= P_R(1+r_f)\mathbb{E}[U'(\tilde{w})] \iff \\ (1-\theta(1-\lambda(S_{ESM})))\,\mathbb{E}\left[U'(\tilde{w})\tilde{F}_G\right] &= P_R(1+r_f)\mathbb{E}[U'(\tilde{w})]\end{aligned}$$

Writing

$$\mathbb{E}\left[U'(\tilde{w})\tilde{F}_G\right] = \text{cov}\left(U'(\tilde{w}), \tilde{F}_G\right) + \mathbb{E}[U'(\tilde{w})]\,\mathbb{E}\left[\tilde{F}_G\right]$$

and applying Stein's Lemma, yields

$$\mathbb{E}\left[U'(\tilde{w})\tilde{F}_G\right] = \mathbb{E}[U''(\tilde{w})]\,\text{cov}\left(\tilde{w}, \tilde{F}_G\right) + \mathbb{E}[U'(\tilde{w})]\,\mathbb{E}\left[\tilde{F}_G\right]$$

Hence:

$$(1-\theta(1-\lambda(S_{ESM})))\left(\mathbb{E}\left[U''(\tilde{w})\,\text{cov}\left(\tilde{w}, \tilde{F}_G\right)\right] + \mathbb{E}[U'(\tilde{w})]\,\mathbb{E}\left[\tilde{F}_G\right]\right) = P_R(1+r_f)\,\mathbb{E}[U'(\tilde{w})]$$

The covariance can be written as:

$$\begin{aligned}\text{cov}\left(\tilde{w}, \tilde{F}_G\right) &= \text{Cov}\left(X_R^\top\left((1-\theta)\tilde{F}_G + \theta\lambda(S_{ESM})\tilde{F}_G\right), \tilde{F}_G\right) \\ &= X_R^\top(1-\theta(1-\lambda(S_{ESM})))\,\Sigma_{F_G}\end{aligned}$$

where $\Sigma_{\tilde{F}_G}$ is the variance-covariance matrix of the pay-off vector $\tilde{F}_G$. Collecting terms yields:

$$\begin{aligned}&(1-\theta(1-\lambda(S_{ESM})))\left(\mathbb{E}[U''(\tilde{w})]\,X_R^\top(1-\theta(1-\lambda(S_{ESM})))\,\Sigma_{F_G} + \mathbb{E}[U'(\tilde{w})]\,\mathbb{E}\left[\tilde{F}_G\right]\right) \\ &= P_R(1+r_f)\,\mathbb{E}[U'(\tilde{w})]\end{aligned}$$

[29] Given two continuously-differentiable stochastic variables, X and Y, that are jointly normal, Stein's lemma states that $\mathbb{C}\text{ov}[f(X), Y] = \mathbb{E}\left[f'(X)\right]\mathbb{C}\text{ov}[X, Y]$.

Hence,

$$P_R = \frac{(1-\theta(1-\lambda(S_{ESM})))}{1+r_f}\left(\mathbb{E}\left[\tilde{F}_G\right] + \frac{\mathbb{E}\left[U''(\tilde{w})\right]}{\mathbb{E}\left[U'(\tilde{w})\right]} X_R^\top (1-\theta(1-\lambda(S_{ESM})))\,\Sigma_{F_G}\right)$$

We assume that $U(\cdot)$ is a function with constant coefficient of absolute risk aversion $\kappa > 0$. Defining $\eta > 0$ as the coefficient of relative risk-aversion, i.e. $W\kappa = \eta$, where W is primary dealer wealth, using that $X_R^\top \Sigma_{\tilde{F}} = \Sigma_{\tilde{F}} X_R$, assuming market clearing $X_R = S_{CapMkt}$, where S_{CapMkt} is the supply of bonds on the capital market, and assuming without loss of generality that $r_f = 0$, we can write:

$$P_R = (1-\theta(1-\lambda(S_{ESM})))\left(\mathbb{E}\left[\tilde{F}_G\right] - \frac{\eta(1-\theta(1-\lambda(S_{ESM})))}{W}\Sigma_{F_G} S_{CapMkt}\right)$$

The novel feature of the model is the supply of the ESM official loans. Whereas the primary dealers optimize utility, the supply of the ESM is assumed to be price-inelastic. The motivation for this assumption is that the objective of its financial assistance programs is not to maximize the expected return (subject to a risk budget). In fact, the ESM is committed to supplying an amount of assets equal to the sovereign's financing need. Denote the size of the ESM's official loan by S_{ESM}. Hence, $S = S_{CapMkt} + S_{ESM}$, and we can write:

$$P_R = (1-\theta(1-\lambda(S_{ESM})))\left(\mathbb{E}\left[\tilde{F}_G\right] - \frac{\eta(1-\theta(1-\lambda(S_{ESM})))}{W}\Sigma_{F_G}(S - S_{ESM})\right)$$

As an example, suppose that following an adverse shock, Spain needs to inject 70 billion euros into the financial sector. Then, S needs to increase by 70 billion. If the ESM provides a loan of 40 billion, the "funding gap" to be placed with primary dealers is 30 billion. The placement puts negative pressure on the price of the bond, hence upward pressure on its yield. Clearly, the official credit line of the ESM dampens the yield increase. More formally, the impact of an official credit line can be evaluated by examining the derivative.

Derivative with respect to S_{ESM}

Let $A = (1 - \theta\,(1 - \lambda\,(S_{ESM})))$. Then,

$$P_R = A\left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right)$$

Taking the derivative with respect to S_{ESM}:

$$\begin{aligned}\frac{\partial P_R}{\partial S_{ESM}} &= \frac{\partial A}{\partial S_{ESM}}\left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right) \\ &\quad + A\frac{\partial}{\partial S_{ESM}}\left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right)\end{aligned}$$

Using,

$$\frac{\partial A}{\partial S_{ESM}} = \theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}$$

we have,

$$\begin{aligned}\frac{\partial}{\partial S_{ESM}}\left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right) &= -\frac{\partial}{\partial S_{ESM}}\left(\frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right) \\ &= -\frac{\eta}{W}\Sigma_{F_G}\left(\frac{\partial A}{\partial S_{ESM}}\,(S - S_{ESM}) - A\right) \\ &= -\frac{\eta}{W}\Sigma_{F_G}\left(\theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}\,(S - S_{ESM}) - A\right)\end{aligned}$$

Hence:

$$\begin{aligned}\frac{\partial P_R}{\partial S_{ESM}} &= \theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}\left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta A}{W}\Sigma_{F_G}\,(S - S_{ESM})\right) \\ &\quad - \frac{\eta A}{W}\Sigma_{F_G}\left(\theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}\,(S - S_{ESM}) - A\right) \\ &= \theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}\mathbb{E}\left[\tilde{F_G}\right] - \frac{2\eta\theta\frac{d\lambda(S_{ESM})}{dS_{ESM}}A}{W}\Sigma_{F_G}(S - S_{ESM}) + \frac{\eta A^2}{W}\Sigma_{F_G}\end{aligned}$$

Substituting back $A = (1 - \theta(1 - \lambda(S_{ESM})))$:

$$\begin{aligned}\frac{\partial P_R}{\partial S_{ESM}} &= \theta \frac{d\lambda(S_{ESM})}{dS_{ESM}} \mathbb{E}\left[\tilde{F_G}\right] \\ &- \frac{2\eta\theta \frac{d\lambda(S_{ESM})}{dS_{ESM}} (1 - \theta(1 - \lambda(S_{ESM})))}{W} \Sigma_{F_G}(S - S_{ESM}) \\ &+ \frac{\eta(1 - \theta(1 - \lambda(S_{ESM})))^2}{W} \Sigma_{F_G}\end{aligned}$$

The first term represents the effect of the ESM loan on the recovery value to the primary dealer in the event of a bad shock. The sign of the derivative of $\lambda(\cdot)$ determines the outcome. If the market perceives ESM debt as senior to the other debt, this derivative will be negative, which implies that an increase in the ESM loan pushes up the yield on the debt placed on the capital market. However, when the recovery value increases with the size of the official loan - potentially due to freed-up fiscal space - the aforementioned derivative would be positive, and this term raises the price at which the debt is placed on the market.

The second term concerns the change in risk-bearing capacity due to an adjustment in the downside risk. ESM debt influences the variance of pay-outs by adjusting the recovery rate $\lambda(\cdot)$. Since the primary dealers are risk-averse, this adjustment results in different pricing of the debt. Assuming, as before, that the loan negatively affects the recovery rate, thereby reducing the variance of the pay-out, this will have a price-increasing and therefore yield-decreasing effect. The opposite is the case when the ESM loan raises the recovery rate.

The third term reflects the relative reduction in the amount issued compared to a scenario in which the sovereign must entirely fulfill its financing needs through the capital market. This reduction exerts an upward pressure on bond prices.

Derivative with respect to θ

$$P_R = (1 - \theta(1 - \lambda(S_{ESM}))) \left(\mathbb{E}\left[\tilde{F_G}\right] - \frac{\eta(1 - \theta(1 - \lambda(S_{ESM})))}{W} \Sigma_{F_G}(S - S_{ESM}) \right)$$

Taking the derivative with respect to θ:

$$\begin{aligned}\frac{\partial P_R}{\partial \theta} &= -\left(1-\lambda\left(S_{ESM}\right)\right)\left(\mathbb{E}\left[\tilde{F_G}\right]-\frac{\eta\left(1-\theta\left(1-\lambda\left(S_{ESM}\right)\right)\right)}{W}\Sigma_{F_G}\left(S-S_{ESM}\right)\right)\\ &+\left(1-\theta\left(1-\lambda\left(S_{ESM}\right)\right)\right)\left(\frac{\eta(1-\lambda\left(S_{ESM}\right))}{W}\Sigma_{F_G}\left(S-S_{ESM}\right)\right)\\ &= -\left(1-\lambda\left(S_{ESM}\right)\right)\left(\mathbb{E}\left[\tilde{F_G}\right]-2\frac{\eta\left(1-\theta\left(1-\lambda\left(S_{ESM}\right)\right)\right)}{W}\Sigma_{F_G}\left(S-S_{ESM}\right)\right)\end{aligned}$$

The first term after the first equality sign represents the effect of the probability of the adverse state on gross returns. If the recovery rate is equal to one, this term disappears, as there is no adjustment during a downturn. A lower recovery rate amplifies the negative impact of the probability of the adverse state on prices.

The second term pertains to the change in risk-bearing capacity due to the change in the probability of a bad state. If the recovery rate is one, this term vanishes. Defaults reduce both the first and second moments of payoffs. Therefore, a recovery rate lower than one diminishes uncertainty about the return, exerting upward pressure on the price of the bond.

Appendix C Additional tables

Table 3: Spanish ESM Programme Loan Repayments.

Date	Amount (€blns.)	Cum. amount (€blns.)	Type
08/07/2014	1.304	1.304	voluntary
23/07/2014	0.308	1.612	scheduled (unused funds)
17/03/2015	1.5	3.112	voluntary
14/07/2015	2.5	5.612	voluntary
11/11/2016	1	6.612	voluntary
14/06/2017	1	7.612	voluntary
16/11/2017	2	9.612	voluntary
23/02/2018	2	11.612	voluntary
23/05/2018	3	14.612	voluntary
16/10/2018	3	17.612	voluntary
12/12/2022	3.643	21.255	scheduled
11/12/2023	3.643	24.898	scheduled

Appendix D Additional figures

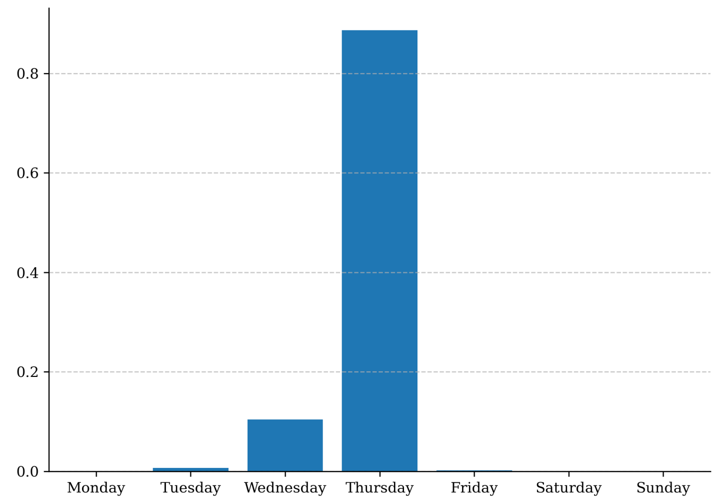


Figure 22: Allocation of auctions over weekdays, fractions.

Figure 23: Total Number of Auctions by Month.

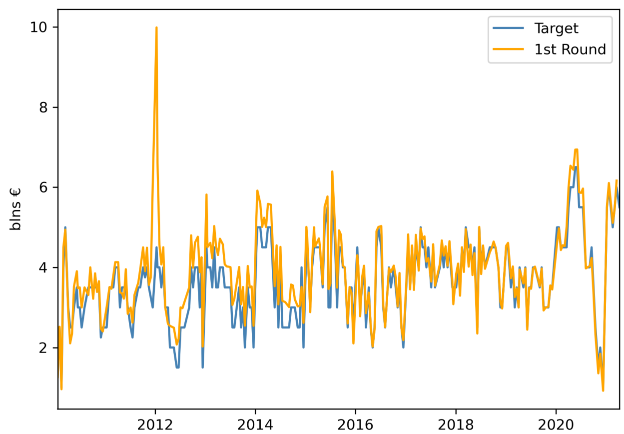


Figure 24: Total targets versus average amount accepted, per auction day.

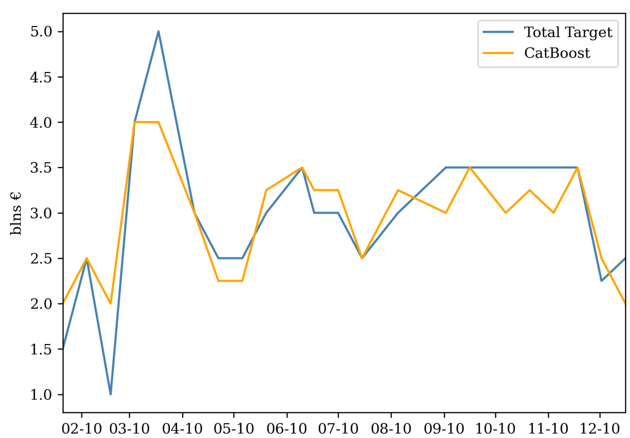


Figure 25: Total targets, backcasted, 2010 validation sample.

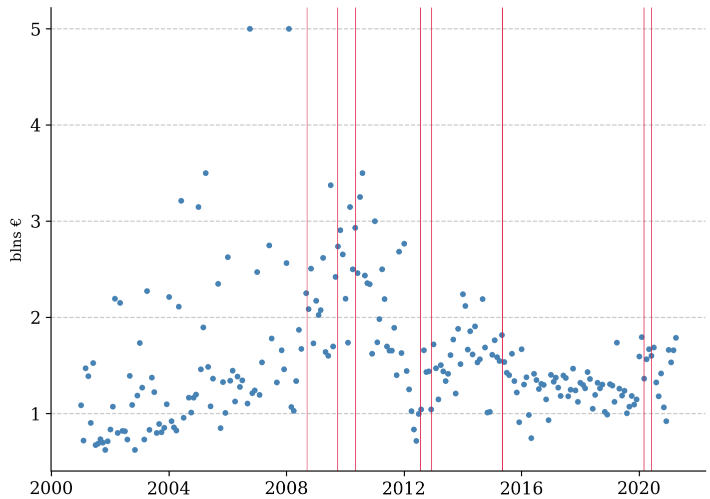


Figure 26: Average amount accepted per issue.

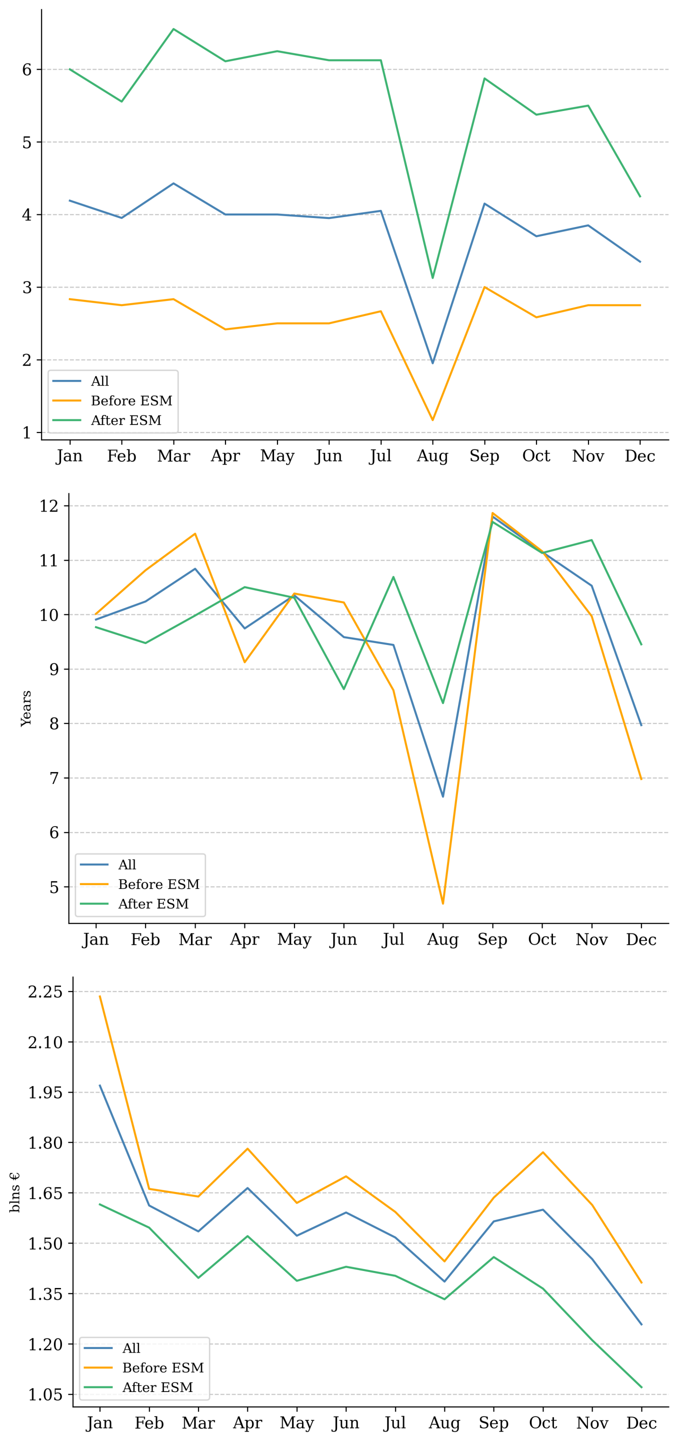


Figure 27: Issue characteristics by month within the calendar year.

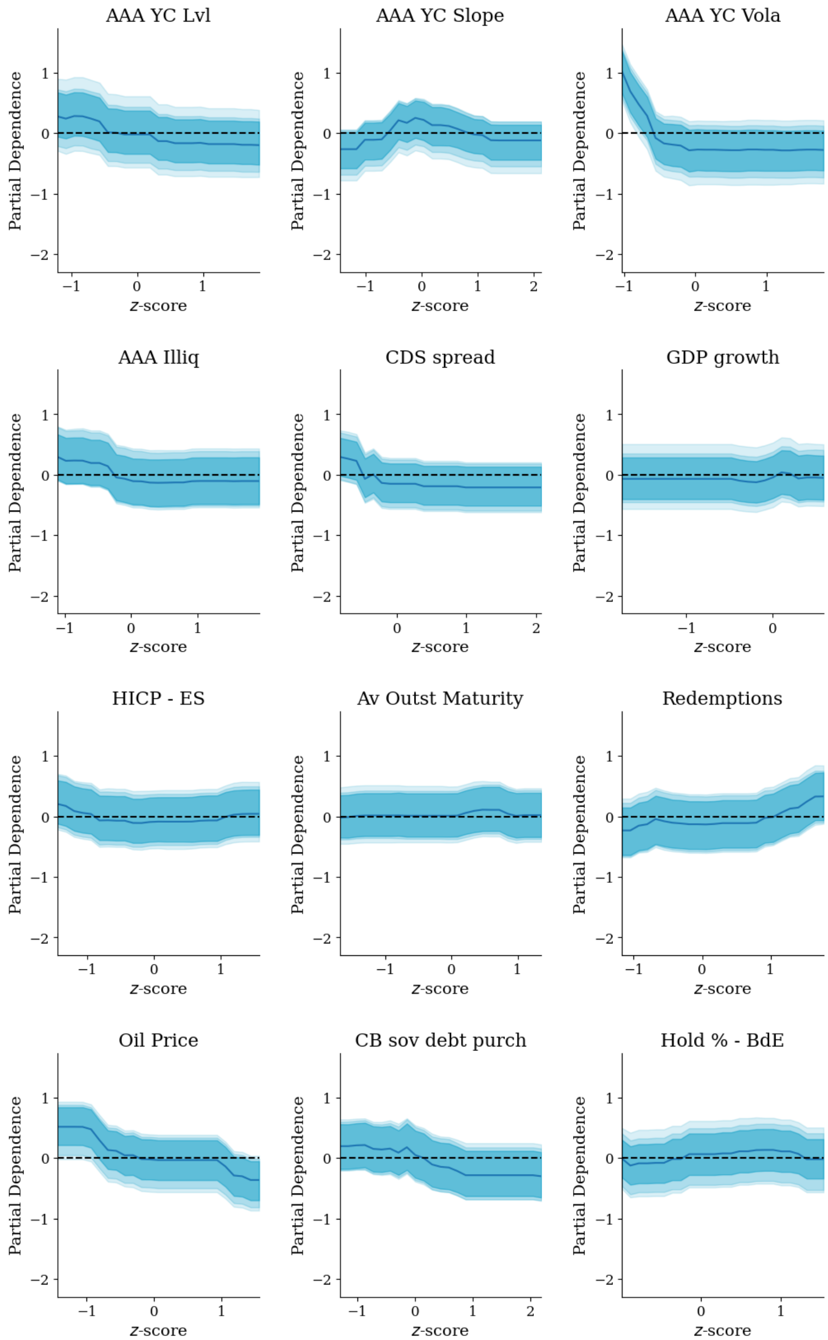


Figure 28: Partial dependence plots of market and economic covariates, effect relative to average maturity.

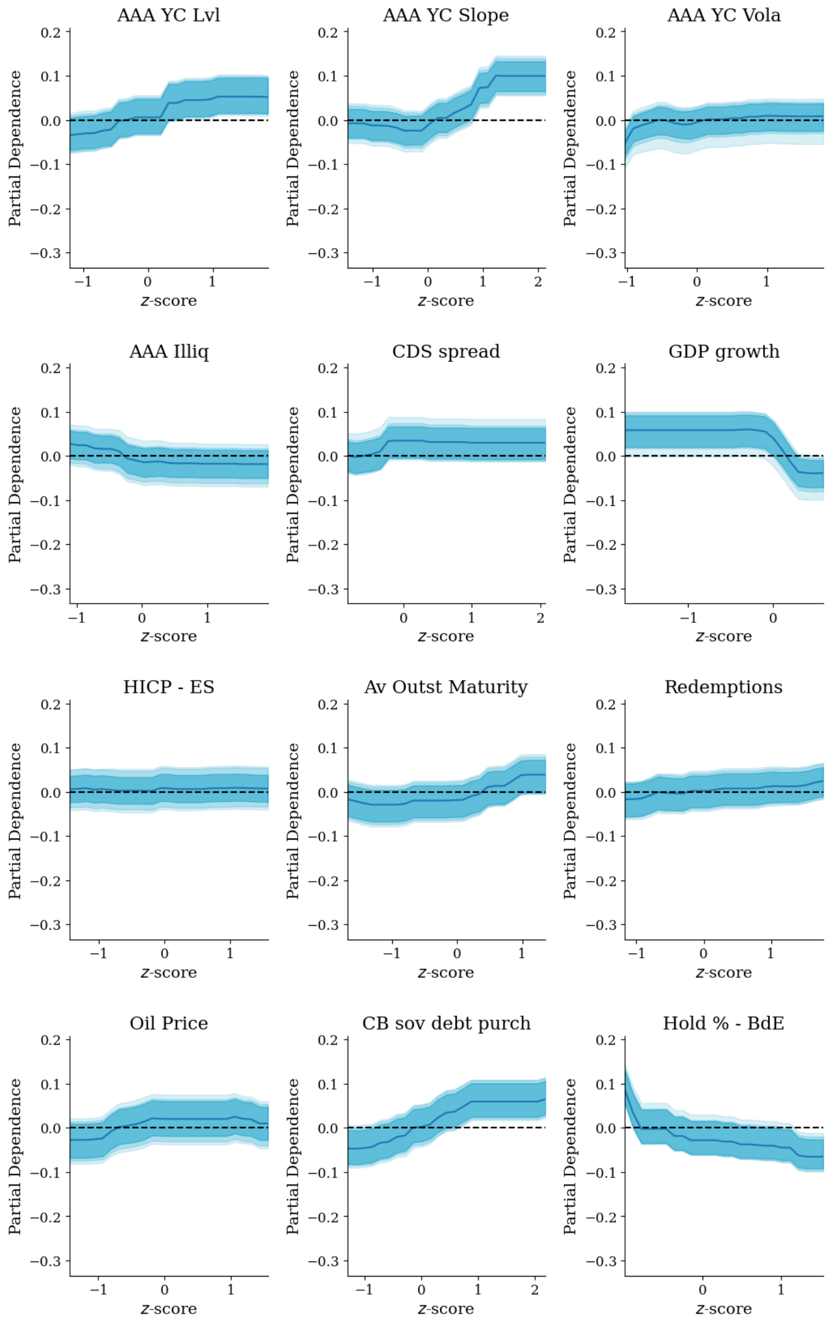
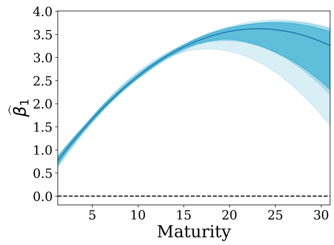
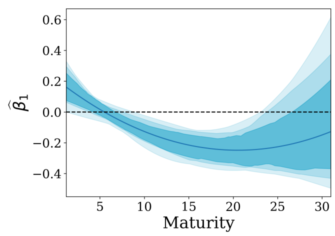


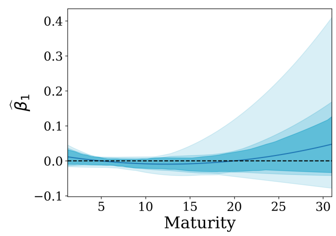
Figure 29: Partial dependence plots of market and economic covariates, effect relative to average target.



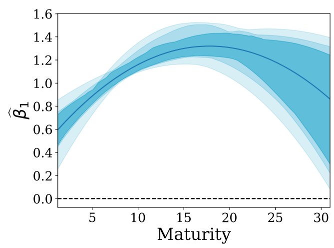
(a) Intercept



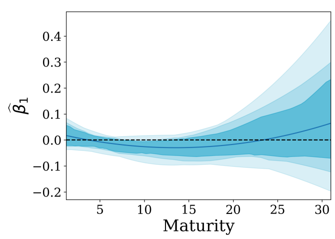
(b) Av Outst Maturity



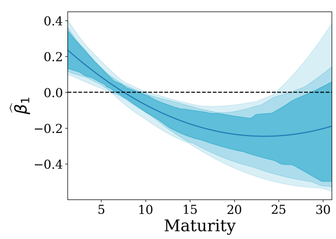
(c) Redemptions



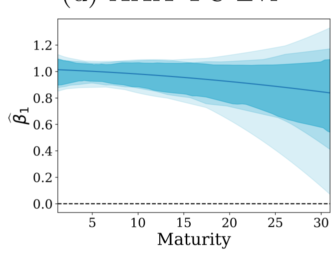
(d) AAA YC Lvl



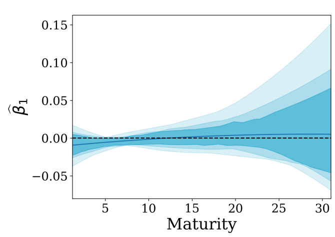
(e) AAA YC Vola



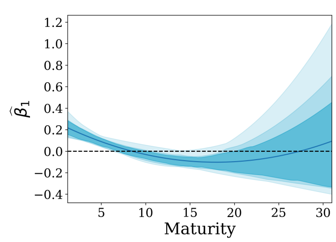
(f) AAA YC Slope



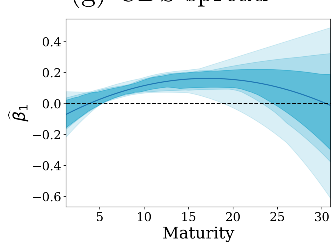
(g) CDS spread



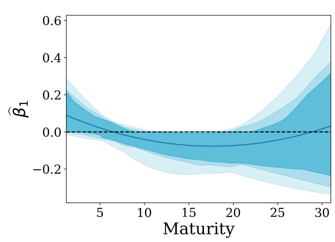
(h) GDP growth



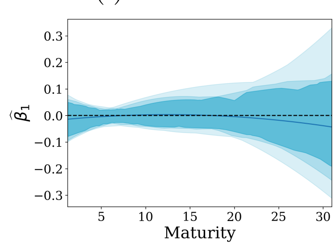
(i) Oil Price



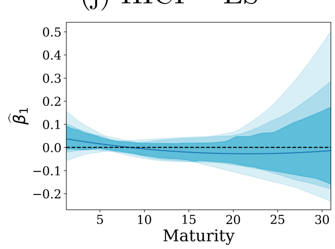
(j) HICP - ES



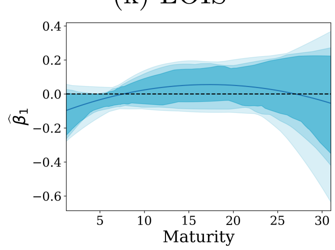
(k) LOIS



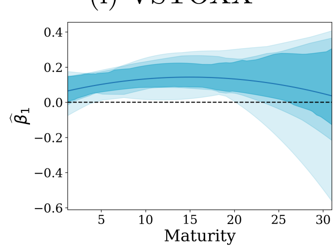
(l) VSTOXX



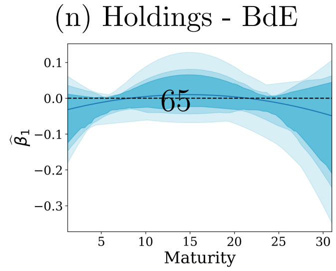
(m) CB sov debt purch



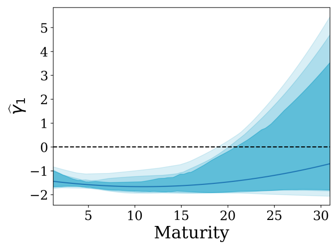
(n) Holdings - BdE



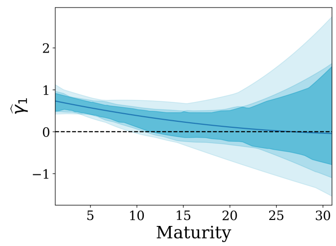
(o) ESM debt relief



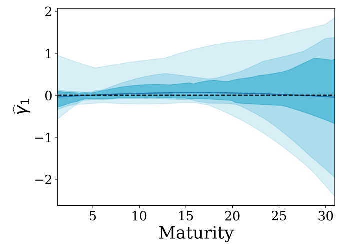
(p) D Month 8



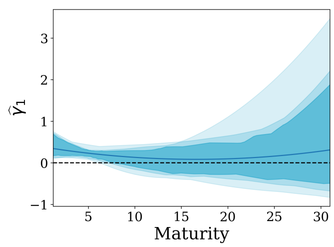
(a) Intercept



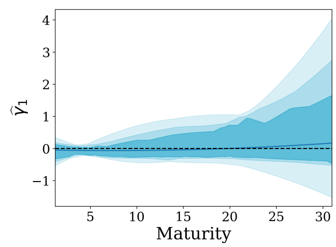
(b) AAA YC Lvl



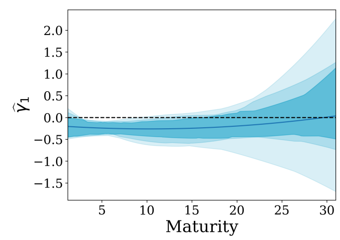
(c) AAA YC Vola



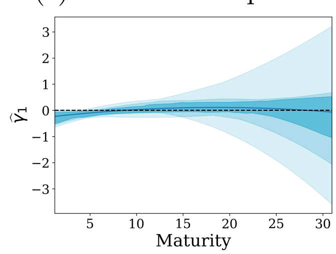
(d) AAA Illiq



(e) CB sov debt purch



(f) Holdings - BdE



(g) ESM debt relief

Figure 31: Robustness, volatility effects $\widehat{\gamma}$

Appendix E Timing of primary market auctions

To formally examine the determinants of auction dates, we employ a Bayesian binary choice model. Here, Y_t, where $t \in [1, 2, \ldots, T]$, represents the binary stochastic variable determining whether an auction occurs on trading day t, with outcomes denoted as y_t, where $t \in [1, 2, \ldots, T]$.[30] We define $\boldsymbol{x}_t$ as a vector comprising predetermined covariates on day t, including lagged dependent variables. The inclusion of lagged dependent variables is essential as the DMO's decision to schedule an auction on day t may hinge on the time elapsed since the last auction day. The probability of at least one auction occurring, denoted as $y_t = 1$, is represented by p_t. Formally, the binary choice model is given by:

$$\begin{aligned} &Y_t \sim Bernoulli(p_t) \\ &p_t = g^{-1}(\boldsymbol{x}_t'\boldsymbol{\beta}) \\ &\beta \sim \pi\left(\beta\right) \end{aligned} \tag{2}$$

where $g(\cdot)$ is the link-function The key assumption for the probit specification of the binary choice model is $g^{-1}(\cdot) = \Phi(\cdot)$, where $\Phi(\cdot)$ is the Gaussian cumulative distribution function. The posterior distributions are given in Appendix E.1.

Table 4 reports the estimates of this dynamic probit model. In view of the frequency plot of the intervals between subsequent auction days in Figure 7, we initially include only an intercept and lags 5, 10, 15, 20, 25, and 30 of the dependent variable, and a dummy for the month August in the set of covariates. We obtain 51000 posterior draws, and let our burn-in and thinning factor be 1000 and 10, respectively, to effectively retain 5000 posterior draws. Column [1] is generally in line with Figure 7. Only the fifth lag is both negative and significant, indicating that if one business week ago an auction day occurred, this makes it less likely that today is also an auction day. Indeed, recall Figure 7, which shows that in less than 10 percent of the cases a new auction takes place one business week after the previous auction. The August dummy is significantly negative in all regression specifications and, therefore, retained in the ensuing regression stages.

[30] Since y_t is 1 or 0, $\sum_{t=1}^{T} y_t = N$, with N the number of auction dates in our sample.

By adding additional control variables, discussed in Subsection 4.2, we try to capture factors determining potential flexibility in the choice of the auction days.[31] The estimates are reported in Columns [1] to [7] in Table 4. Note that we do not control for primary market covariates, as these variables are unknown at the moment the decision is made to call an auction day. Controlling for other covariates does not affect the coefficient estimates of the variables included in the baseline specification.

Despite the model's predictive accuracy of approximately 94%, it struggles to accurately forecast auction dates, with a success rate of only about 49%. This discrepancy can be attributed to the highly imbalanced nature of our dataset. Although such high predictability might typically suggest potential endogeneity issues, this appears improbable in our case. Even the simplest model, as shown in Column [1], demonstrates substantial predictive capability, thereby mitigating concerns regarding endogeneity. Despite the resilience of strong learners to resampling, and particularly to correct probability calibration,[32] as noted by Elor and Averbuch-Elor (2022), we conduct a robustness exercise involving downsampling.[33] In this exercise, we bootstrap samples comprising all auction days alongside an equal number of non-auction days to create a balanced dataset. Remarkably, the results of these bootstrap experiments align with the unsampled results, consistent with the findings of Elor and Averbuch-Elor (2022).

[31] Trying to capture the clustering of auction dates, i.e. the fact that we occasionally observe auctions in two consecutive weeks, we also tried to control for (lagged) Spanish holidays. If the "regular" bi-weekly auction day is a holiday, the auction could be shifted by one week, which reduces the time between two ensuing auction dates to one week only. However, the number of such cases is too low to yield any statistical power.

[32] Calibration refers to the accuracy of predicted probabilities. In a well-calibrated classifier, a predicted probability of, say, 49%, should correspond to a true outcome roughly 49% of the time. This means that if one groups all instances in which the model predicts a 49% probability, one would expect around 49% of them to match the actual outcome. It is important to note that classifiers are not always calibrated by default. Sampling techniques that alter the distribution of target classes can lead to a loss of calibration in the model.

[33] Note that due to undersampling the majority class, the amount of information available to the model is actually reduced.

Table 4: Bayesian dynamic probit results for whether a business day is an auction day.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Intercept	−2.17	−2.18	−2.17	−2.17	−2.19	−2.01	−2.17
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.16)	(0.05)
L5	−1.49	−1.50	−1.49	−1.48	−1.49	−1.50	−1.48
	(0.15)	(0.14)	(0.14)	(0.15)	(0.15)	(0.15)	(0.15)
L10	1.01	1.02	1.01	1.01	1.02	1.05	1.00
	(0.09)	(0.10)	(0.09)	(0.10)	(0.09)	(0.10)	(0.10)
L15	0.35	0.36	0.34	0.36	0.37	0.38	0.35
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)
L20	1.70	1.71	1.70	1.70	1.71	1.71	1.70
	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)	(0.09)
L25	1.85	1.87	1.86	1.85	1.88	1.88	1.86
	(0.12)	(0.11)	(0.12)	(0.12)	(0.11)	(0.12)	(0.11)
L30	0.25	0.25	0.25	0.25	0.28	0.27	0.26
	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
D August	−0.74	−0.73	−0.75	−0.74	−0.77	−0.72	−0.74
	(0.17)	(0.17)	(0.17)	(0.16)	(0.16)	(0.16)	(0.17)
Secondary Market	N	Y	N	N	N	N	N
Primary Dealer	N	N	Y	N	N	N	N
Economic Outlook	N	N	N	Y	N	N	N
Fiscal Policy	N	N	N	N	Y	N	N
Monetary Policy	N	N	N	N	N	Y	N
Official Loans	N	N	N	N	N	N	Y
N	5305	5305	5305	5305	5305	5305	5305
TP (%)	49.26	47.55	48.04	49.02	45.59	46.08	49.26
TN (%)	97.64	97.74	97.64	97.74	97.80	97.78	97.64
Accuracy (%)	93.90	93.86	93.80	93.97	93.76	93.78	93.90

Notes: (i) standard errors are given in parentheses. (ii) "D August" is a dummy for the month August, "TPs" gives percentage correct predictions of an auction day (true positives), and "Correct 0s" gives the percentage of correct predictions of no auction day (true negatives).

To examine if the determination of auction dates involves nonlinear relationships with the covariates, including potential interactions between lagged variables, we create a gradient-boosted ensemble with CatBoost. This algorithm controls for all covariates used in the individual Bayesian probit regressions. For more details on CatBoost, refer to Footnote [15]. Hyperparameters are optimized using Bayesian optimization (Mockus, 1989) of the 5-fold cross-validated ROC AUC. The prediction accuracy increases to 98% due to the increase in the fraction of true positives of the auction dates to 91%.

Figure 32 illustrates the partial dependence plots of the lagged dependent variables, aligning with the results from the Bayesian probit analysis. More details on the partial dependence plots can be found in Footnote [19]. Additionally, Figure 33 depicts the marginal effects of the market and economic variables. Similar to the linear model, these variables are not useful in predicting auction dates. As the relevant variables, the lagged dependent variables, are dummies, the increased accuracy observed in the CatBoost model is attributed to intricate interactions among the lagged dependent variables, which is not captured by the linear model. We view the high predictability of our estimated model as a sign that the DMO has succeeded in its mission to be predictable, albeit in a nonlinear fashion.

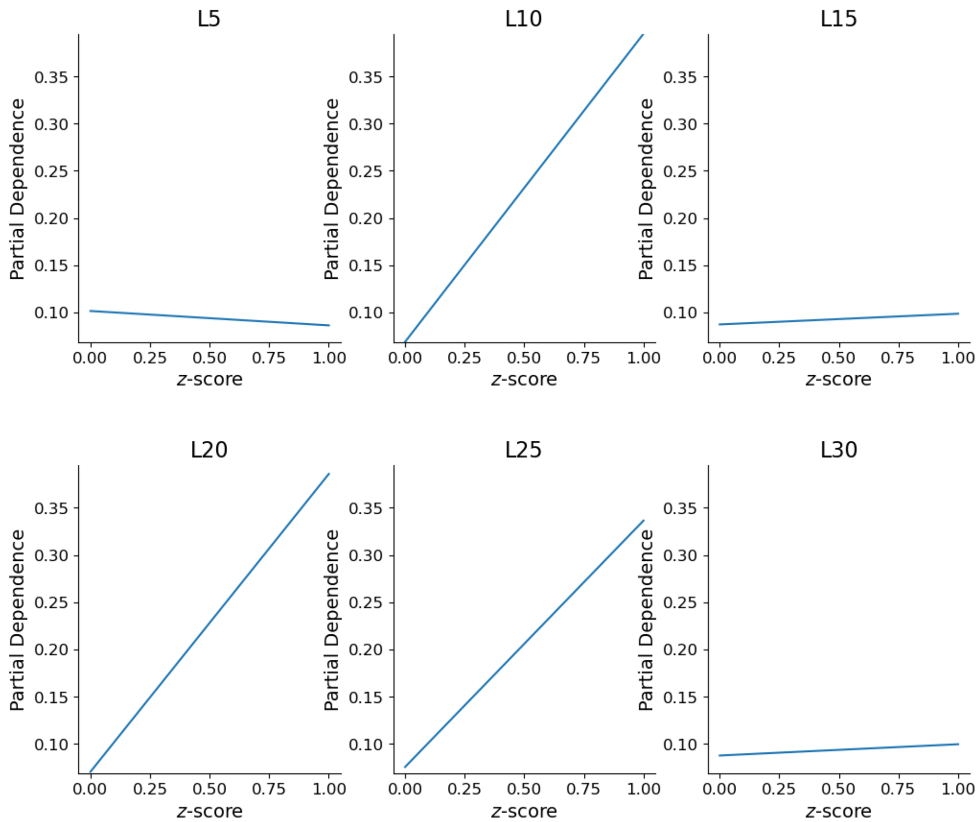


Figure 32: Auction date determination partial dependence of lagged dependent variables.

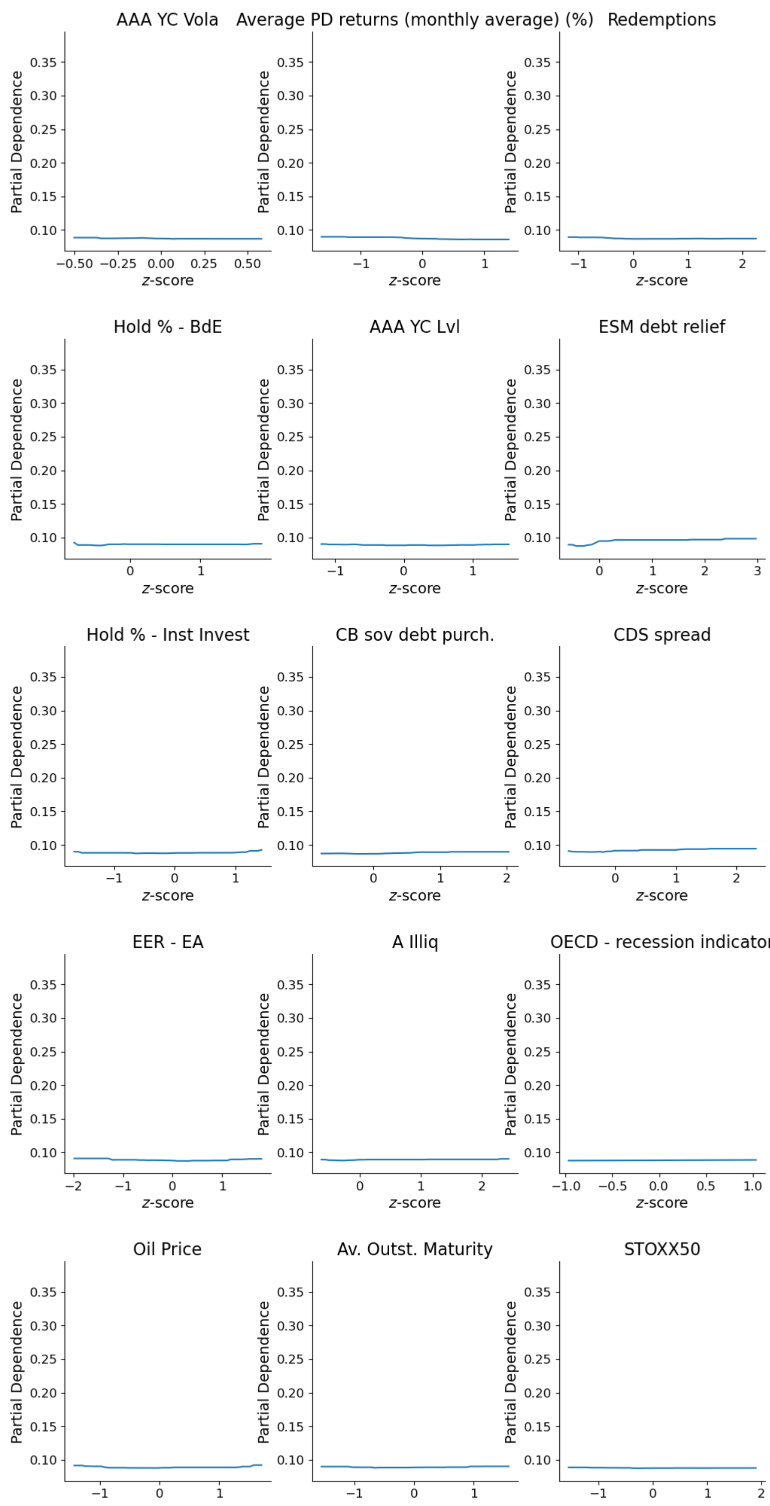


Figure 33: Auction date determination partial dependence of market and economic covariates.

E.1 Bayesian probit posterior distributions

The joint probability distribution of $y = (y_1, y_2, \ldots, y_T)'$, the likelihood, is:

$$\mathbb{P}(Y = y|\beta) = \prod_{t=1}^{T} \Phi(x_t'\beta)^{y_t} \left[1 - \Phi(x_t'\beta)\right]^{1-y_t} \tag{3}$$

Following Albert and Chib (1993), we introduce T latent variables $Z_t \sim \mathcal{N}(x_t'\beta, 1)$, $t \in [1, 2, \ldots, T]$,[34] and let $y_t = \mathbb{1}[Z_t > 0]$, where $\mathbb{1}[\cdot]$ is the indicator function which equals one if the expression in brackets holds and zero, otherwise. With a Gibbs sampler one is able to simulate β from the exact posterior density. The joint posterior density of β and $Z = (Z_1, \ldots, Z_T)'$ given data y is given by:

$$\begin{aligned} p(\beta, Z|y) &\propto p(y|\beta, Z)\pi(\beta) \\ &= \pi(\beta) \prod_{t=1}^{T} \left(\mathbb{1}[Z_t > 0]\,\mathbb{1}[y_t = 1] + \mathbb{1}[Z_t \leq 0]\,\mathbb{1}[y_t = 0]\right) \phi\left(Z_t - x_t'\beta\right) \end{aligned} \tag{4}$$

where $\phi(\cdot)$ is the standard normal density. The posterior density of β yields:

$$p(\beta|y, Z) \propto \pi(\beta) \prod_{t=1}^{T} (Z_t - x_t'\beta) \tag{5}$$

Hence, the posterior density is that of the parameter in the $Z = X'\beta + \varepsilon$ regression, with $\varepsilon \sim \mathcal{N}(0, I)$, I the identity matrix, and $X = (x_1, \ldots, x_T)'$. As such, given the conjugate prior $\mathcal{N}(b_0, B_0)$, the posterior distribution of β is given by:

$$\begin{aligned} \beta|y, Z &\sim \mathcal{N}(b, B) \\ b &= B\left(B_0^{-1} b_0 + X'Z\right) \\ B &= \left(B_0^{-1} + X'X\right)^{-1} \end{aligned} \tag{6}$$

As the Z_t are independent random variables, the posterior distribution of Z given β reads:

$$Z_t|y, \beta \propto \begin{cases} \mathcal{N}(x_t'\beta, 1)\mathbb{1}[Z_t > 0], & \text{if } y_t = 1 \\ \mathcal{N}(x_t'\beta, 1)\mathbb{1}[Z_t \leq 0], & \text{otherwise} \end{cases} \tag{7}$$

[34] The variance of the latent variables is normalized to unity for identification purposes.

However, Holmes and Held (2006) state that a potential problem in the model arises due to the strong posterior correlation between β and Z. Moreover, this correlation is likely to cause slow convergence to the steady state distribution. They suggest to sample β and Z jointly by factorizing $p(b, Z|y) = p(\beta|Z)p(Z|y)$. Note that the sampling of β is just as in (6). However, Z is updated from its marginal distribution instead of its conditional. Moreover, as sampling from a truncated multivariate normal is notoriously difficult, each element of Z_t is updated conditionally on the current other elements. Holmes and Held (2006) derive, assuming a zero-mean normal prior for β ($b_0 = 0$):

$$Z_t|Z_{-t}, y_t \sim \begin{cases} \mathcal{N}(m_t, v_t)\mathbb{1}\left[Z_t > 0\right], & \text{if } y_t = 1 \\ \mathcal{N}(m_t, v_t)\mathbb{1}\left[Z_t \leq 0\right], & \text{otherwise} \end{cases} \tag{8}$$

where Z_{-t} denotes the latent variable vector omitting Z_t, and:

$$\begin{aligned} m_t &= x_t'b - w_t(z_t - x_t'b) \\ v_t &= 1 + w_t \\ w_t &= \frac{h_t}{1 - h_t} \\ h_t &= (H)_{ii} \end{aligned} \tag{9}$$

where $H = XBX'$. Following an update of each Z_t, the posterior mean of β needs to be recalculated:

$$b = b^{old} + S_t(Z_t - Z_t^{old}) \tag{10}$$

where S_t denotes the t^{th} column of $S = BX'$ and the superscript *old* indicates the values before the update.

Appendix F Number of issues per auction day

Denote the number of issues on auction day t by N_t. We model N_t using a CatBoost classifier, using the variables in section 4.2, and month and year dummies as covariates. For more details on CatBoost, refer to footnote [15]. Hyperparameters are optimized using Bayesian optimization (Mockus, 1989) of the 5-fold cross-validated ROC AUC. The optimized CatBoost model encompasses 170 trees with a maximum depth of 4. At each level it samples 90% of the covariates, and learns with 15%. Each leaf contains a minimum of 25 observations.

The cross-validated confusion matrix depicted in Figure 34 showcases the model's performance. Overall, the prediction accuracy is very high at 96%. The model faces the largest challenges in accurately predicting auction days with two and four issues. Nevertheless, the model still achieves approximately 91% and 93% accuracy in predicting the number of auctions during these auction days, respectively. This difficulty partly stems from the distribution of auction days across the sample period: days with two issuances are spread throughout the dataset, while days with one issuance are more concentrated in the early part and those with three or four issuances are concentrated towards the end. The infrequent occurrence of days with four issuances is concentrated towards the end of the sample, see Figure 8.

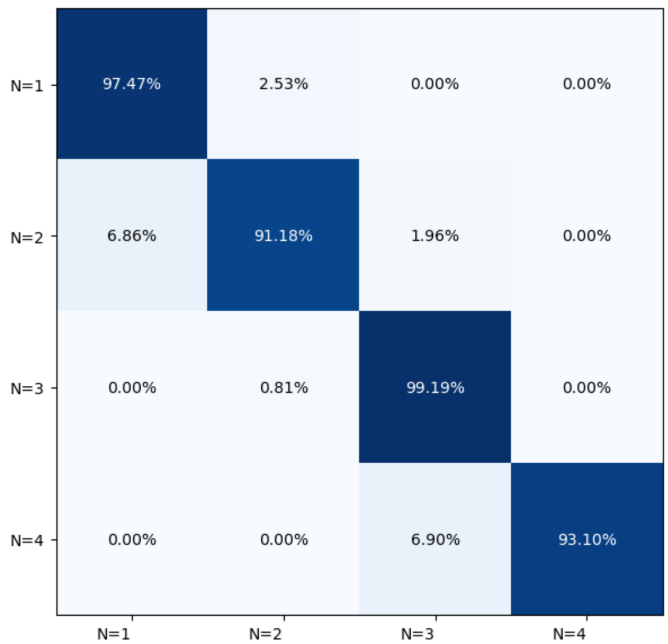


Figure 34: Cross-validated confusion chart

Figure 35 presents the partial dependence plots for the nine most critical features. Note that the curves always sum up to one for all z-scores. The targeted amount on the auction day emerges as the most informative, with higher values contributing to increased probabilities of multiple issue days. Strong nonlinear patterns are evident across all four curves. On single-issue auction days, there is a rapid decline in probability. Additionally, the modes of the distributions for multiple-issue days are sequentially ordered, each following the decrease in probability of the preceding auction day type.

During periods of turmoil, characterized by increased Spanish government debt and additional liquidity injections from the ECB, the DMO tended to issue more debt across multiple auctions. The holdings of sovereign debt by the Banco de España, or the central bank stock effect, appear to influence the shift from two-issue to three-issue auction days, alongside a decrease in one-issue and four-issue days.

A similar interchange is observed with changes in the debt-to-GDP ratio, where one- and three-issue auction days swap positions, while two-issue days decrease as the ratio rises.

Although other variables exhibit less significance, a consistent pattern emerges: one-issue auction days are most frequently observed, followed by three-, two-, and four-issue days, in that order, across all z-score values.

Interestingly, the presence of ESM official loans does not appear to significantly impact the DMO's issue strategy in terms of the number of issues on a particular auction day.

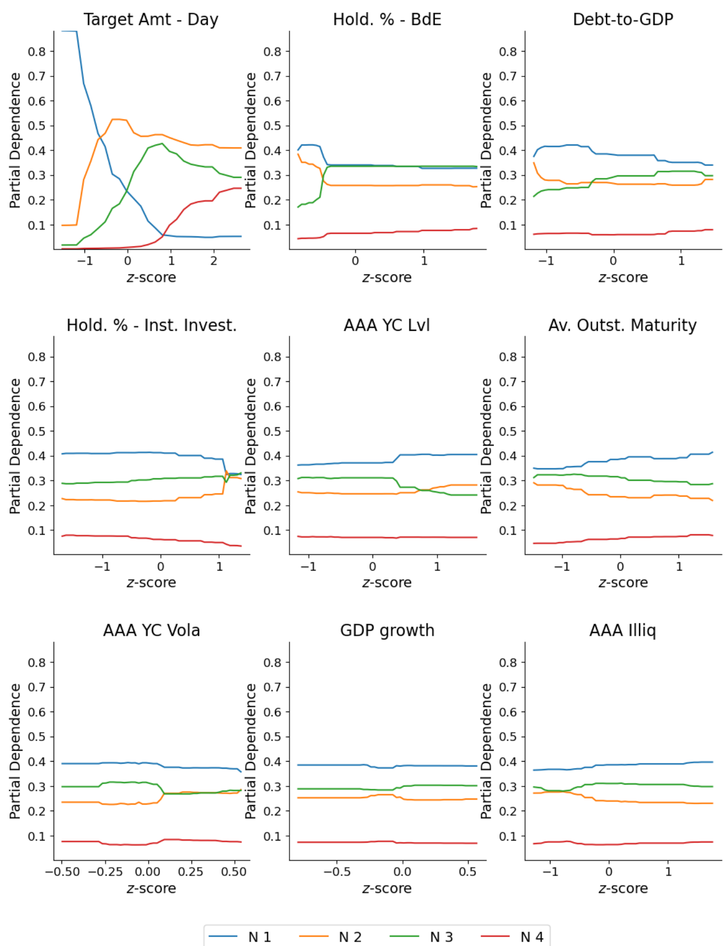


Figure 35: Number of auctions partial dependence plots